ОЛІГОПОЛІЯ ТА ІНШІ РИНКИ НЕДОСКОНАЛОЇ КОНКУРЕНЦІЇ

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COURNOT COMPETITION YIELDS SPATIAL DISPERSION

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Abstract

Introduction. *A lot of works suggests that Cournot oligopolists competing in a spatial model, with a uniform distribution of consumers, agglomerate in the center of the market. In this paper revisited some results from [1]. In the paper [1] showed that Cournot-type oligopolists which discriminate over space will tend to agglomerate. The paper [2] considers the spatial model used by [2] to study firms' decisions on locations without restricting the consumers' reservation price.* ***Purpose.*** *This paper extend the analysis of the standard model of spatial discrimination with Cournot competition along the linear city for a high enough transport tariff.* ***Results.*** *It was obtained that for a high enough transport tariff the firms have a decision which lies on the boundary of the feasible locations region. We show that a change in the central agglomeration strategy to the dispersion strategy occurs at the point of transcritical bifurcation. The different effects come into play. Before bifurcation point the effect of minimizing transport costs is dominate. Firms choose the central agglomeration strategy to minimize a total distance of transportation. The growth of the transport tariff leads to a decrease in the total profit. In the bifurcation point begins to dominate the effect of market segmentation. Firms choose a dispersed strategy to monopolize adjacent markets. The growth of the transport tariff leads to an increase in total profits. The growth of total profit with growth of the transport tariff is due to the fact that when dispersion strategy, the firms supply more to adjoining markets and less to distant markets. In the case of multiple equilibria is shown that exactly the stable solution provides a large profit. The conditions for full coverage of the markets for both strategies are defined.* ***Conclusions.*** *In this paper we show that firms under Cournot competition will tend to dispersion. Thus, the article extends the analysis of the standard Hotelling spatial competition model. The results allow a deeper look at the causes of agglomeration and dispersion of firms. The analysis of equilibrium stability showed that the transport tariff is a bifurcation parameter for firms when choosing a spatial strategy.*

Key words: *linear city, Cournot competition, agglomeration, dispersion, transcritical bifurcation.*

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КОНКУРЕНЦІЯ КУРНО ПРИЗВОДИТЬ ДО ПРОСТОРОВОЇ ДИФЕРЕНЦІАЦІЇ

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Анотація

Вступ. *У багатьох роботах було отримано, що олігополісти Курно, конкуруючі в просторовій моделі з рівномірним розподілом споживачів, будуть агломеруватися в центрі ринку. У цій статті розвиваються деякі результати з [1, 2]. В роботі [1] показано, що олігополісти Курно, які здійснюють просторову дискримінацію, будуть прагнути до центральної агломерації. В роботі [2], на основі моделі з [1], аналізуються оптимальні просторові рішення фірм для більш високої ціни попиту.* ***Мета.*** *Дана стаття розширює аналіз стандартної моделі просторової дискримінації з конкуренцією Курно уздовж лінійного міста на випадок більш високого транспортного тарифу.* ***Результати.*** *Було отримано, що для досить високого транспортного тарифу у фірм існує рішення, яке лежить на границі області можливих місць розташування. Обгрунтовано, що зміна стратегії центральної агломерації на стратегію диференціації відбувається в точці транскрітичної біфуркації. Показано, що у точці біфуркації взаємодіють різні ефекти. До точки біфуркації домінує ефект мінімізації транспортних витрат. Фірми вибирають стратегію центральної агломерації, щоб мінімізувати загальну відстань транспортування. Зростання транспортного тарифу призводить до зниження загального прибутку. У точці біфуркації починає домінувати ефект сегментації ринків. Фірми вибирають стратегію диференціації для монополізації прилеглих ринків. Подальше зростання транспортного тарифу призводить до збільшення загального прибутку. Зростання загального прибутку з ростом транспортного тарифу пов'язано з тим, що при стратегії диференціації фірми більше постачають на сусідні ринки, а менше – на віддалені. При існуванні декількох рівноваг показано, що саме стійке рівновага забезпечує більший прибуток. Також визначено умови повного охоплення ринків для обох стратегій.* ***Висновки.*** *У цій статті ми показуємо, що фірми в умовах конкуренції Курно будуть прагнути до диференціації. Таким чином, стаття розширює аналіз стандартної моделі просторової конкуренції Хотеллінга. Отримані результати дозволяють глибше поглянути на причини агломерації та диференціації фірм. Аналіз стійкості рівноваги показав, що транспортний тариф є біфуркаційним параметром для фірм при виборі просторової стратегії.*

Ключові слова: *лінійне місто, конкуренція Курно, агломерація, диференціація, транскрітична біфуркація.*

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КОНКУРЕНЦИЯ КУРНО ПРИВОДИТ К ПРОСТРАНСТВЕННОЙ ДИФФЕРЕНЦИАЦИИ

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Аннотация

Введение. *Во многих работах было получено, что олигополисты Курно, конкурирующие в пространственной модели с равномерным распределением потребителей, будут агломерироваться в центре рынка. В этой статье развиваются некоторые результаты из [1, 2]. В работе [1] показано, что олигополисты Курно, осуществляющие пространственную дискриминацию, будут стремиться к центральной агломерации. В работе [2], на основе модели из [1], анализируются оптимальные пространственные решения фирм для более высокой цены спроса.* ***Цель.*** *Данная статья расширяет анализ стандартной модели пространственной дискриминации с конкуренцией Курно вдоль линейного города на случай более высокого транспортного тарифа.* ***Результаты.*** *Было получено, что для достаточно высокого транспортного тарифа у фирм существует решение, которое лежит на границе области возможных местоположений. Обосновано, что изменение стратегии центральной агломерации на стратегию дифференциации происходит в точке транскритической бифуркации. Показано, что в точке бифуркации взаимодействуют различные эффекты. До точки бифуркации доминирует эффект минимизации транспортных расходов. Фирмы выбирают стратегию центральной агломерации, чтобы минимизировать общее расстояние транспортировки. Рост транспортного тарифа приводит к снижению общей прибыли. В точке бифуркации начинает преобладать эффект сегментации рынков. Фирмы выбирают стратегию дифференциации для монополизации близлежащих рынков. Дальнейший рост транспортного тарифа приводит к увеличению общей прибыли. Рост общей прибыли с ростом транспортного тарифа связан с тем, что при стратегии дифференциации фирмы больше поставляют на соседние рынки, а меньше – на отдаленные. В случае нескольких равновесий показано, что именно устойчивое решение обеспечивает бóльшую прибыль. Также определены условия полного охвата рынков для обеих стратегий.* ***Выводы.*** *В этой статье мы показываем, что фирмы в условиях конкуренции Курно будут стремиться к дифференциации. Таким образом, статья расширяет анализ стандартной модели пространственной конкуренции Хотеллинга. Полученные результаты позволяют глубже взглянуть на причины агломерации и дифференциации фирм. Анализ устойчивости равновесия показал, что транспортный тариф является бифуркационным параметром для фирм при выборе пространственной стратегии.*

Ключевые слова: *линейный город, конкуренция Курно, агломерация, дифференциация, транскритическая бифуркация.*

Introduction and problem statement

In search of a solution to the Bertrand paradox, Hotelling proposed to take into account the factor of space under the price competition of firms. In Hotelling's linear city model [3], two firms compete on a segment with a unit demand at each point. Firms optimize their prices and location on the segment. Transportation delivery costs of goods are borne by consumers. Hotelling found that in an equilibrium state, firms would be minimally spatially differentiated, since they would be located in the center. This conclusion of the model analysis subsequently became a famous "principle of minimal differentiation".

In further research, the Hotelling model has developed in the following areas:

– an increase in the number of firms [4, 5];

– increase the dimension of space [6, 7];

– the complexity of the type of transport costs function [8, 9];

– generalization of the consumer's distribution density [10, 11, 12];

– consideration of the Cournot competition [1, 2, 13, 14, 15] and Stackelberg competition [16, 17, 18].

Anderson and Neven [1] restricted the analysis to $t < b/2$. Rivas [2] extend the analysis to $t \le b$ allowing for different market configurations. The paper identified market patterns where firms compete over the whole market as well as patterns where a firm behaves as a monopoly in a market segment.

Formulation of article goals. In this paper we are extending the analysis to $t \le 2 \cdot b$ and showing that firms have location decisions which provide a full markets cover.

Description of the main research material

1. The linear city model

Two firms sell homogeneous goods on the unit segment, at each point of which is the consumer market x, $x \in [0, 1]$. The distance of the firms from zero point is equal x_1 and x_2 accordingly, and $x_1 \le x_2$. Each firm faces linear transportation costs of t to move one good unit per one unit of distance. Consumer arbitrage is assumed to be prohibitively costly.

The linear demand curve in the market x:

$$p(x) = b - q_1(x) - q_2(x),$$

where $p(x)$ – the price in the market x, $q_1(x)$, $q_2(x)$ – the quantities supplied of firms in the market x, b – a minimum price, at which there is no demand (reservation price).

Let us assume that firms supply products to all markets (full coverage): $q_1(x = 1) \ge 0$, $q_1(x < 1) > 0$, $q_2(x = 0) \ge 0$, $q_2(x > 0) > 0$. Thus, zero quantities supplied are possible only at the boundaries of a unit segment.

The profits of firms in the market x:

$$F_1(x) = q_1(x)\cdot\left(b - q_1(x) - q_2(x) - t\cdot|x - x_1|\right) \to \max_{x_1, q_1(x)},$$

$$F_2(x) = q_2(x)\cdot\left(b - q_1(x) - q_2(x) - t\cdot|x - x_2|\right) \to \max_{x_2, q_2(x)}.$$

The competitive game consists of two stages. In the first stage, the firms simultaneously select their locations. In the second stage, at the given location decisions, the firms simultaneously choose their supplied quantities. The equilibrium of the model is solved by backward induction.

2. The Cournot competition

According to the backward induction method we begin with the second stage. Let us assume that firms optimize supply volumes under Cournot competition. Solving the first-order conditions yields the reaction curves of the firms:

$$q_1(x) = \frac{b - q_2(x) - t\cdot|x - x_1|}{2}, \quad q_2(x) = \frac{b - q_1(x) - t\cdot|x - x_2|}{2}.$$

The equilibrium supply volumes of firms to the market x:

$$q_1^*(x) = \frac{b - 2\cdot t\cdot|x - x_1| + t\cdot|x - x_2|}{3}, \tag{1}$$

$$q_2^*(x) = \frac{b - 2\cdot t\cdot|x - x_2| + t\cdot|x - x_1|}{3}. \tag{2}$$

Let us define the feasible locations region of firms.

1. From previous studies [2, 11, 13, 15] we know that the equilibrium in this model is symmetrical about the center:

$$x_1^e + x_2^e = 1, \; x_1^e \le 1/2, \; x_2^e \ge 1/2. \tag{3}$$

2. In the center of line segment the firms minimize a total distance of traffic, therefore full markets coverage is possible with a highest transport tariff. Substituting into (1) the values $x_1 = 1/2$, $x_2 = 1/2$, $x = 1$ or into (2) the values $x_1 = 1/2$, $x_2 = 1/2$, $x = 0$, we find that at any locations of firms the coverage of all markets is possible only at $t \le 2\cdot b$.

3. From (1) it follows that for firm 1 the minimum volume of deliveries is reaching in the market $x = 1$. Therefore a condition of markets coverage for firm 1:

$$q_1^*(x = 1) = 0 \quad \Leftrightarrow \quad b - 2\cdot t\cdot(1 - x_1) + t\cdot(1 - x_2) = 0. \tag{4}$$

For firm 2 the minimum volume of deliveries is reaching in the market $x = 0$. Therefore a condition of markets coverage for firm 2:

$$q_2^*(x = 0) = 0 \quad \Leftrightarrow \quad b - 2\cdot t\cdot x_2 + t\cdot x_1 = 0. \tag{5}$$

Solving the system of equations (4)-(5) yields

$$x_1^{cov} = \frac{1}{2} - \frac{2 \cdot b - t}{6 \cdot t}, \quad (6)$$

$$x_2^{cov} = \frac{1}{2} + \frac{2 \cdot b - t}{6 \cdot t}. \quad (7)$$

Thus, the feasible locations region are (Fig.1):

$$\begin{aligned} &0 \le x_1 \le 1/2, \text{ for } 0 < t \le b/2, \\ &x_1^{cov} \le x_1 \le 1/2, \text{ for } b/2 < t \le 2 \cdot b, \\ &1/2 \le x_2 \le 1, \text{ for } 0 < t \le b/2, \\ &1/2 \le x_2 \le x_2^{cov}, \text{ for } b/2 < t \le 2 \cdot b. \end{aligned} \quad (8)$$

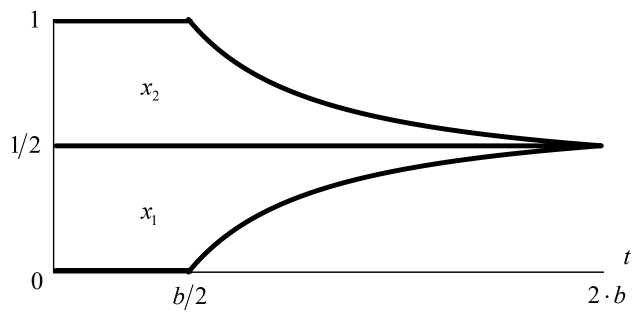

Figure 1. The feasible locations region. Source: Own elaboration

The equilibrium profits of firms in the market x :

$$\begin{aligned} F_1^*(x) &= \frac{\left(b - 2 \cdot t \cdot |x - x_1| + t \cdot |x - x_2|\right)^2}{9} = \left(q_1^*(x)\right)^2, \\ F_2^*(x) &= \frac{\left(b - 2 \cdot t \cdot |x - x_2| + t \cdot |x - x_1|\right)^2}{9} = \left(q_2^*(x)\right)^2. \end{aligned} \quad (9)$$

In the first stage each firm selects a profit-maximizing location at a given location of the rival.

So, let us start with firm 1. The total profit of firm 1 in all markets:

$$F_1 = \int_0^1 F_1^*(x)\,dx = \int_0^{x_1} F_1^*(x)\,dx + \int_{x_1}^{x_2} F_1^*(x)\,dx + \int_{x_2}^{1} F_1^*(x)\,dx,$$

$$\begin{aligned} 9 \cdot F_1 = &\int_0^{x_1} \left(b + 2 \cdot t \cdot (x - x_1) - t \cdot (x - x_2)\right)^2 dx + \\ &+ \int_{x_1}^{x_2} \left(b - 2 \cdot t \cdot (x - x_1) - t \cdot (x - x_2)\right)^2 dx + \\ &+ \int_{x_2}^{1} \left(b - 2 \cdot t \cdot (x - x_1) + t \cdot (x - x_2)\right)^2 dx. \end{aligned} \quad (10)$$

After integrating and identical transformations (10), we obtain:

$$81 \cdot \mathrm{t} \cdot \mathrm{F}_1 = 4 \cdot \left(b - t \cdot (x_1 - x_2)\right)^3 + 2 \cdot \left(b + 2 \cdot t \cdot (x_1 - x_2)\right)^3 -$$

$$-3 \cdot \left(b - t \cdot (2 \cdot x_1 - x_2)\right)^3 - 3 \cdot \left(b + t \cdot (2 \cdot x_1 - x_2 - 1)\right)^3 .$$

The optimal location is defined by the necessary condition:

$$\frac{9}{4 \cdot \mathrm{t}} \cdot \frac{\partial \mathrm{F}_1}{\partial x_1} = \mathrm{t} \cdot (x_1 - x_2)^2 + 2 \cdot b \cdot (x_1 - x_2) - (2 \cdot b - t) \cdot (2 \cdot x_1 - x_2 - 1/2) = 0 . \qquad (11)$$

The sufficient condition for the existence of profit maximum for the firm 1:

$$\frac{9}{8 \cdot \mathrm{t}} \cdot \frac{\partial^2 \mathrm{F}_1}{\partial x_1^2} = -\,\mathrm{t} \cdot (\mathrm{x}_2 - \mathrm{x}_1) - (b - t) < 0 .$$

The necessary condition for the existence of the equilibrium location for firm 1 is the nonnegativity of the discriminant of the square equation (11):

$$\mathrm{D}_1 = 4 \cdot (b - t)^2 + 4 \cdot t \cdot (2 \cdot b - t) \cdot (x_2 - 1/2) \geq 0 . \qquad (12)$$

It is easy to make sure that $\mathrm{D}_1 > 0$ at $\mathrm{x}_2 \geq 1/2$. Therefore, due to condition (3), in the equilibrium state the discriminant (12) is always nonnegative.

The roots of the square equation (11) are:

$$\left(\mathrm{x}_1^*\right)_1 = \mathrm{x}_2 + \frac{b - t}{\mathrm{t}} - \frac{\sqrt{\mathrm{D}_1}}{2 \cdot \mathrm{t}} , \quad \left(\mathrm{x}_1^*\right)_2 = x_2 + \frac{b - t}{\mathrm{t}} + \frac{\sqrt{\mathrm{D}_1}}{2 \cdot \mathrm{t}} .$$

The root $\left(\mathrm{x}_1^*\right)_2$ does not satisfy the basic conditions of the model and therefore is not further analyzed. The total profit of firm 2 in all markets:

$$\mathrm{F}_2 = \int_0^1 \mathrm{F}_2^*(\mathrm{x})\, dx = \int_0^{\mathrm{x}_1} \mathrm{F}_2^*(\mathrm{x})\, dx + \int_{\mathrm{x}_1}^{\mathrm{x}_2} \mathrm{F}_2^*(\mathrm{x})\, dx + \int_{\mathrm{x}_2}^{1} \mathrm{F}_2^*(\mathrm{x})\, dx .$$

$$9 \cdot \mathrm{F}_2 = \int_0^{\mathrm{x}_1} \left(b + 2 \cdot t \cdot (x - x_2) - t \cdot (x - x_1)\right)^2 dx +$$

$$+\int_{\mathrm{x}_1}^{\mathrm{x}_2} \left(b + 2 \cdot t \cdot (x - x_2) + t \cdot (x - x_1)\right)^2 dx +$$

$$+\int_{\mathrm{x}_2}^{1} \left(b - 2 \cdot t \cdot (x - x_2) + t \cdot (x - x_1)\right)^2 dx . \qquad (13)$$

After integrating and identical transformations (13), we obtain:

$$81 \cdot \mathrm{t} \cdot \mathrm{F}_2 = 4 \cdot \left(b + t \cdot (x_2 - x_1)\right)^3 + 2 \cdot \left(b - 2 \cdot t \cdot (x_2 - x_1)\right)^3 -$$

$$-3 \cdot \left(b - t \cdot (2 \cdot x_2 - x_1)\right)^3 - 3 \cdot \left(b + t \cdot (2 \cdot x_2 - x_1 - 1)\right)^3 .$$

The optimal location is defined by the necessary condition:

$$\frac{9}{4 \cdot t} \cdot \frac{\partial F_2}{\partial x_2} = 2 \cdot b \cdot (x_2 - x_1) - t \cdot (x_2 - x_1)^2 - (2 \cdot b - t) \cdot (2 \cdot x_2 - x_1 - 1/2) = 0 . \quad (14)$$

The sufficient condition for the existence of profit maximum for the firm 2:

$$\frac{9}{8 \cdot t} \cdot \frac{\partial^2 F_2}{\partial x_2^2} = - t \cdot (x_2 - x_1) - (b - t) < 0 .$$

The necessary condition for the existence of the equilibrium location for firm 2 is the nonnegativity of the discriminant of the square equation (14):

$$D_2 = 4 \cdot (b - t)^2 + 4 \cdot t \cdot (2 \cdot b - t) \cdot (1/2 - x_1) \geq 0 . \quad (15)$$

It is easy to make sure that $D_2 > 0$ at $x_1 \leq 1/2$. Therefore, due to condition (3), in the equilibrium state, the discriminant (15) is always nonnegative. The roots of the square equation (14) are:

$$\left(x_2^*\right)_1 = x_1 - \frac{b - t}{t} + \frac{\sqrt{D_2}}{2 \cdot t} , \quad \left(x_2^*\right)_2 = x_1 - \frac{b - t}{t} - \frac{\sqrt{D_2}}{2 \cdot t} .$$

The root $\left(x_2^*\right)_2$ does not satisfy the basic conditions of the model and therefore is not further analyzed. Thus, we received the reaction curves of firms:

$$x_1 = x_2 + \frac{b - t}{t} - \frac{\sqrt{(b - t)^2 + t \cdot (2 \cdot b - t) \cdot (x_2 - 1/2)}}{t} , \quad (15)$$

$$x_2 = x_1 - \frac{b - t}{t} + \frac{\sqrt{(b - t)^2 + t \cdot (2 \cdot b - t) \cdot (1/2 - x_1)}}{t} . \quad (16)$$

Substituting (15) into (16), we are obtaining the symmetry condition (3). Using the symmetry condition (3), we find solutions of the system (15)-(16):

$$x_1^{agg} = x_2^{agg} = 1/2 , \quad (17)$$

$$x_1^{dis} = 1/2 - \frac{3 \cdot t - 2 \cdot b}{4 \cdot t} , \quad (18)$$

$$x_2^{dis} = 1/2 + \frac{3 \cdot t - 2 \cdot b}{4 \cdot t} . \quad (19)$$

So, we have obtained two equilibrium location strategies for firms: central agglomeration and symmetric dispersion. For $t = 2 \cdot b/3$, the solutions (17) and (18)-(19) coincide. From the location condition, $x_1 \leq x_2$, it follows that firms can apply the dispersion strategy only when $t \geq 2 \cdot b/3$.

3. The analysis of the stability of equilibrium

Let us analyze a stability of the solutions (17)-(19). For this we consider a two-dimensional map:

$$\begin{aligned} x_1(n+1) &= x_2(n) + \frac{b-t}{t} - \frac{\sqrt{(b-t)^2 + t\cdot(2\cdot b - t)\cdot(x_2(n) - 1/2)}}{t}, \\ x_2(n+1) &= x_1(n) - \frac{b-t}{t} + \frac{\sqrt{(b-t)^2 + t\cdot(2\cdot b - t)\cdot(1/2 - x_1(n))}}{t}, \end{aligned} \tag{20}$$

where n is a time moment, $n = 0, 1, 2, ...,$ $x_1(0) = 0$, $x_2(0) = 1$.

The nature of the stability of fixed points is determined by their multipliers. The multipliers are eigenvalues of the Jacobian matrix in a fixed point, and their number is equal to the dimension of map.

The Jacobian matrix of the map (20) in the fixed point (17):

$$J = \begin{pmatrix} 0 & -\dfrac{t}{2\cdot(b-t)} \\ -\dfrac{t}{2\cdot(b-t)} & 0 \end{pmatrix}. \tag{21}$$

From (21) we obtain two real multipliers:

$$\mu_{1,2} = \pm\frac{t}{2\cdot(b-t)}. \tag{22}$$

For $|\mu_{1,2}| < 1$ the fixed point is stable, for $|\mu_{1,2}| > 1$ the fixed point is unstable, for $|\mu_{1,2}| = 1$ the bifurcation occurs. From (22) it follows that the fixed point (17) is stable when $t < 2\cdot b/3$ and is unstable when $t > 2\cdot b/3$. The loss of stability occurs at the bifurcation point: $t = 2\cdot b/3$.

The Jacobian matrix of the map (20) in a fixed point (18)-(19):

$$J = \begin{pmatrix} 0 & -\dfrac{2\cdot(b-t)}{t} \\ -\dfrac{2\cdot(b-t)}{t} & 0 \end{pmatrix}. \tag{23}$$

From (23) we obtain two real multipliers:

$$\mu_{1,2} = \pm\frac{2\cdot(b-t)}{t}. \tag{24}$$

From (24) it follows that the fixed point (18)-(19) is unstable when $t < 2\cdot b/3$ and is stable when $t > 2\cdot b/3$. The acquisition of stability occurs at the bifurcation point: $t = 2\cdot b/3$.

So we can summarize results in

Proposition 1. At the value of the transport tariff $t = 2 \cdot b/3$ occurs a transcritical bifurcation, in which the spatial strategies exchange of stabilities.

The transcritical bifurcation diagram for $b = 1$ depicted in Fig. 2. The dynamics of the total profit of firm 1 at crossing of the bifurcation point for $b = 1$ depicted in Figure 3.

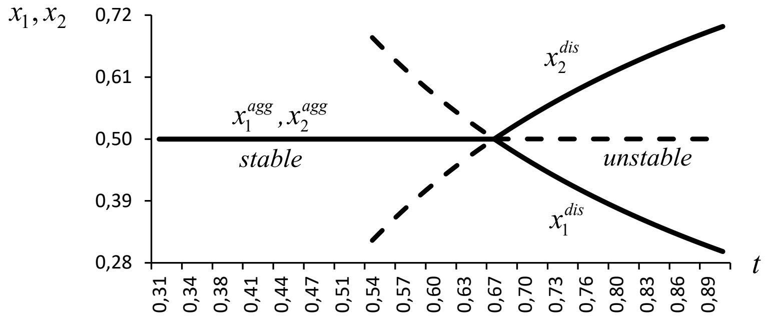

Figure 2. Transcritical bifurcation diagram of map (20). Source: Own elaboration

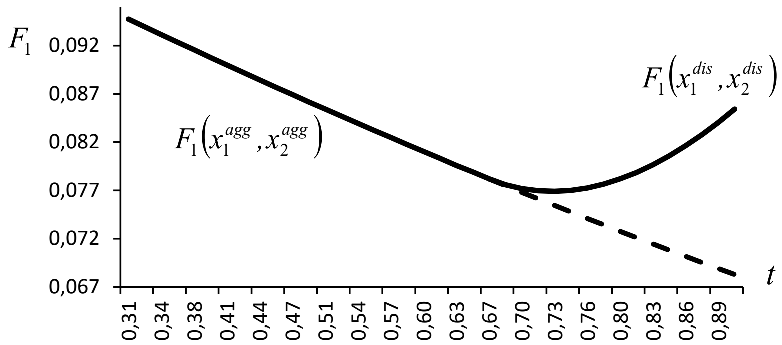

Figure 3. Dynamics of the total profit of firm 1. Source: Own elaboration

In Fig. 3 we see that in the case of multiple equilibria (18)-(20), exactly the stable solution provides a large profit (Fig. 3). The Fig. 3 illustrates the effects that affect spatial strategies of firms. Before bifurcation point the effect of minimizing transport costs is dominate [14]. Firms choose the central agglomeration strategy to minimize a total distance of transportation. The growth of the transport tariff leads to a decrease in the total profit. In the bifurcation point begins to dominate the effect of market segmentation. Firms choose a dispersed strategy to monopolize adjacent markets. The growth of the transport tariff leads to an increase in total profits. The growth of total profit with growth of the transport tariff is due to the fact that when dispersion strategy, the firms supply more to adjoining markets and less to distant markets.

Note that the equilibrium profits of firms (4) are squares of supply volumes and, thus, “ignore” their negative values. For this reason, dispersion strategies (18)-(19) do not take into account restrictions on full market coverage (4)-(5). Solving the systems of equations (4) and (18), (5) and (19), we find that dispersion strategies (18)-(19) are defined only for $t \le 10 \cdot b/11$.

A value of $t^{cov} = 10 \cdot b/11$ was first obtained in [2]. At the point $t^{cov} = 10 \cdot b/11$, the potential for further differentiation of firms is exhausted. For $10 \cdot b/11 < t \le 2 \cdot b$ several solutions are possible. Rivas [2] considered a case when each firm monopolizes a segment on the boundaries of the market and competes with the rival firm in the rest choosing separated locations. Firms symmetrically refuse to cover all markets and this seems like an implicit collusion. In the future such pattern may leads to separation of the unit interval on the two monopoly segments. Subject to continued coverage of all markets in [2] proposed a central agglomeration. However, there is a better solution presented in

Proposition 2. For $10 \cdot b/11 < t \le 2 \cdot b$ the equilibrium spatial strategies lie on the boundary of the feasible locations region.

To provide full markets cover when $10 \cdot b/11 < t \le 2 \cdot b$, firms optimize location based on condition (8), i.e. seek the conditional profit maximum.

The equilibrium spatial strategies and total profits of firms for $0 < t \le 2 \cdot b$ and $b = 1$ depicted in Fig.4 and Fig.5. In the Fig.5 we see that for $10 \cdot b/11 < t \le 2 \cdot b$ the central agglomeration is a worst decision.

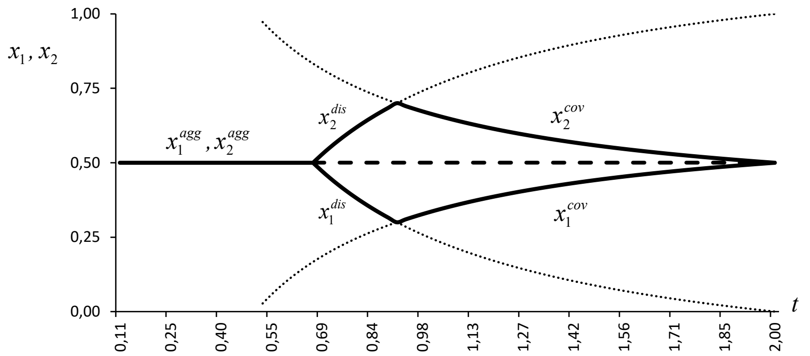

Figure 4. Equilibrium spatial strategies of firms. Source: Own elaboration

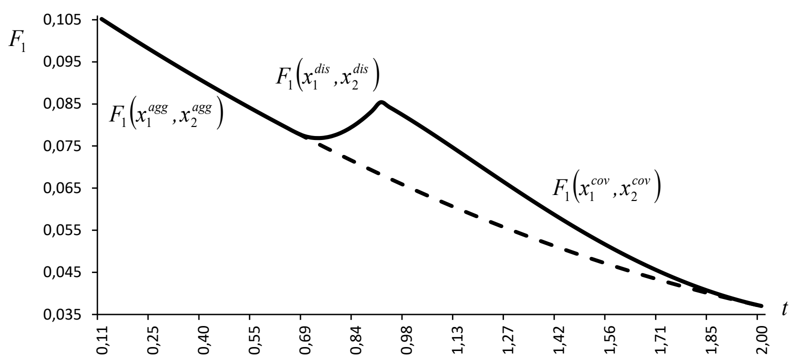

Figure 5. Total profit. Source: Own elaboration

Conclusions

We generalize Rivas [2], Anderson and Neven's analysis [1] by considering a broader interval of the transport tariff. The solution says that when $0 < t \le 10 \cdot b/11$ we still replicate the previous results. For $0 < t \le 2 \cdot b/3$, firms locate at the center, for $2 \cdot b/3 < t \le 10 \cdot b/11$ there are a multiple equilibria: a dispersed equilibrium together with the agglomerated one obtained before. Subject to continued coverage of all markets for $10 \cdot b/11 < t \le 2 \cdot b$ the equilibrium spatial strategies lie on the boundary of the feasible locations region. In the process of the analysis of equilibrium stability, it is proved that the transport tariff is a bifurcation parameter for firms. It has shown that a change in the central agglomeration strategy to the dispersion strategy occurs at the point of transcritical bifurcation. The different effects come into play. Before bifurcation point the effect of minimizing transport costs is dominate. Firms choose the central agglomeration strategy to minimize a total distance of transportation. The growth of the transport tariff leads to a decrease in the total profit. In the bifurcation point begins to dominate the effect of market segmentation. Firms choose a dispersed strategy to monopolize adjacent markets. The growth of the transport tariff leads to an increase in total profits. The growth of total profit with growth of the transport tariff is due to the fact that when dispersion strategy, the firms supply more to adjoining markets and less to distant markets.

The purpose of further research is to analyze the competitive interaction of firms in the Hotelling's linear city model under the conditions of other equilibrium types.

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