FEATURES OF DEFORMING OF COMPOSITE INDUCTORS DURING ELECTROMAGNETIC PROCESSING OF MATERIALS

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Summary

Introduction. The development of many branches of mechanical engineering at the current stage requires the development and application of new technologies that are characterized by high energy efficiency and a small amount of waste. Such technologies include those that use electromagnetic field energy. The energy of the electromagnetic field can be used to change the shape of workpieces, to connect individual structural elements (for example, by welding), to control the physical properties of the material (for example, by induction heating), etc. Carrying out calculation studies at the stage of designing and proving the appropriate technological equipment allows determining the rational values of key parameters, which ensures the achievement of the goals of the technological operation and the operability of the equipment.

Purpose. When the energy of the electromagnetic field is used to exert a force on the processed workpiece, the technological equipment is equally subject to force. The purpose of the article is to create effective methods of calculating the strength of technological equipment together with the workpiece, and conducting the corresponding calculation analysis is an urgent task in the scientific and practical sense.

Results. The article proposes an effective method of analyzing the elastic-plastic deformation of composite structures under the influence of an electromagnetic field. The general mathematical formulation of the coupled problem of the deformation of conductive bodies in the presence of an electromagnetic field is considered. To construct a numerical solution method, the initial problem is reduced to finding the minimum of the total energy of the system. The finite element method is used as a numerical solution method. The proposed method is applied to the
analı́zy of the deformation of the "inductor-workpiece" system of the technological operation of magnetic pulse processing of metals. Conclusions. Some results are presented that allow making certain recommendations regarding the design and application of technological operations of a similar class.

**Key words:** non-stationary deformation, electromagnetic field, the finite el-ement method, magnetic-pulse processing.

**ОСОБЛИВОСТІ ДЕФОРМУВАННЯ СКЛАДЕНИХ ІНДУКТОРИВ ПРИ ЕЛЕКТРОМАГНІТНІЙ ОБРОБЦІ МАТЕРІАЛІВ**

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**Анотація**

**Вступ.** Розвиток багатьох галузей машинобудування на сучасному етапі потребує розробки і застосування нових технологій, які характеризуються високою енергоефективністю та малою кількістю відходів. До таких технологій відносяться ті, що використовують енергію електромагнітного поля. Енергія електромагнітного поля може використовуватись для формо-міні заготовок, для з'єднання окремих елементів конструкції (наприклад шляхом зварювання), для керованої зміни фізичних властивостей матеріалу (наприклад, шляхом індукційного нагріву) тощо. Проведення розрахункових досліджень на етапі проектування та доведення відповідного технологічного оснащення дозволяє визначати раціональні значення ключових параметрів, що забезпечує досягнення цілей технологічної операції та працездатність обладнання. **Мета.** При використанні енергії електромагнітного поля для силового впливу на оброблювану заготовку в рівній мірі силовому впливу піддається і технологічне обладнання. Метою статті є створення ефективних методів розрахунків на міцність технологічного оснащення разом із заготовкою та проведення відповідного розрахункового аналізу з актуальною задачею у науковому та практичному сенсі. **Результати.** У статті запропо-новано ефективний метод аналізу пружно-пластичного деформування скла-денних конструкцій під дією електромагнітного поля. Розглянуто загальну математичну постановку зв’язаної задачі деформування електропровідних тіл при наявності електромагнітного поля. Для побудови методу чисельного розв’язку вихідна задача зведена до пошуку мінімуму нової енергії системи.
В якості чисельного методу розв'язання використовується метод скінчених елементів. Запропонований метод застосований для аналізу деформування системи „індуктор–заготовка“ технологічної операції магнітно-імпульсної обробки металів. Висновки. Представлени дієві результати, які дозволяють робити певні рекомендації щодо проєктування та застосування технологічних операцій подібного класу.

Ключові слова: пружно-пластичне деформування, електромагнітне поле, метод скінчених елементів, магнітно-імпульсна обработка.

Introduction. The development of many branches of mechanical engineering at the current stage requires the development and application of new technologies that are characterized by high energy efficiency and a small amount of waste. Such technologies include those that use electromagnetic field (EM-field) energy. The energy of the EM-field is used for a large number of technological operations. The energy of the electromagnetic field can be used to change the shape of workpieces, to connect individual structural elements (for example, by welding), to control the physical properties of the material (for example, by induction heating), etc. The force effect of EM-field on the processed workpiece is used during magnetic pulse processing of materials (MPPM). Carrying out calculation studies at the stage of designing and proving the appropriate technological equipment allows determining the rational values of key parameters, which ensures the achievement of the goals of the technological operation and the operability of the equipment.

Formulation of the problem. In the case of MPPM, not only the workpiece but also the tool – the inductor – is exposed to electromagnetic forces. An increase in the magnitude of electromagnetic forces leads to more intensive deformation of both the workpiece and the inductor, which can lead to the loss of its efficiency. Therefore, the creation of effective methods for the analysis of elastic-plastic deformation of elements of technological systems of MPPM and subsequent calculation of strength is an urgent scientific and practical problem.

Analysis of recent research and publications. A review of issues related to aspects of the creation and application of devices that use the energy of EM-field for technological purposes is quite fully given in the works [1–4]. Modern directions of development of MPPM for processing non-traditional objects are presented in the works [5, 6]. It should be noted that the majority of scientific publications, for example [7] devoted to the problems of calculation and design of technological operations of MPPM are focused on studies of workpiece deformation. At the same time, the inductor, which also experiences significant mechanical effects, is practically not studied anywhere.

Mathematical formulation of the problem. Let us give a brief mathematical formulation of the problem, based on the general relationships shown in the article [8].

The distribution of the vector components of the EM-field in and electrically conductive body is described by the system of Maxwell’s fundamental equations, which are supplemented by material dependencies and boundary conditions:

\[
\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \vec{j}; \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} - \vec{j} \cdot \vec{H} = 0; \nabla \cdot \vec{E} = 0; \nabla \cdot \vec{H} = \gamma_1 \vec{E} + \gamma_2 \left[ \vec{u} \times \vec{B} \right].
\]  

(1)
\[ \vec{D} = \varepsilon_e \vec{E}; \vec{B} = \mu_e \vec{H}; \vec{E}_\Gamma \times \vec{n} = 0; \quad \vec{D}_\Gamma \cdot \vec{n} = 0; \quad \vec{H}_\Gamma \times \vec{n} = 0; \quad \vec{B}_\Gamma \cdot \vec{n} = 0, \]  

where \( \vec{H}, \vec{E}, \vec{H}_\Gamma, \vec{E}_\Gamma \) are the vectors of the intensity of the magnetic and electric fields in the volume of the body and at the boundary (\( \Gamma \)) of the body; \( \vec{D}, \vec{B}, \vec{D}_\Gamma, \vec{B}_\Gamma \) are the vectors of electric and magnetic induction in the volume of the body and on the boundary (\( \Gamma \)) of the body; \( \varepsilon_e, \mu_e, \gamma \) are the electrical and magnetic permeability, and electrical conductivity of the material; \( \vec{j} \) is the current density vector; \( \vec{n} \) is the vector of normal to the boundary of the body.

The distribution of stress \( \sigma \) and strain \( \varepsilon \) tensor components, as well as the displacement vector \( \vec{u} \), is subjected to a system of equations that contains differential equations of equilibrium, geometric dependencies, material state dependencies, and boundary conditions:

\[ \nabla \times \vec{H} = \varepsilon_e \frac{\partial \vec{E}}{\partial t} + \vec{j} \times \vec{E} = -\mu_e \frac{\partial \vec{H}}{\partial t} + \nabla \times \vec{H} = 0; \quad \nabla \cdot \vec{E} = 0; \quad \vec{j} = \gamma_e \vec{E} + \gamma_c \left( \hat{\vec{u}} \times \vec{B} \right); \]  

where \( \rho \) is the material density; \( \vec{j} \) is the intensity vector of volume forces (in the case of taking into account electromagnetic forces \( \vec{j} = \vec{j} \times \vec{B} \)); \( (4) \) \( \vec{F} \) is the the 4-th-rank correspondence tensor, which in the case of elastic deformation has the form according to Hooke's linear law; \( \vec{p} \) is the surface load; \( \Xi, I \) are the densities of surface charges and currents. When considering elastic-plastic deformation, the behavior of the material is considered in the incremental form (4) together with the condition of plasticity (5):

\[ \nabla \times \vec{H} = \varepsilon_e \frac{\partial \vec{E}}{\partial t} + \vec{j} \times \vec{E} = -\mu_e \frac{\partial \vec{H}}{\partial t} + \nabla \times \vec{H} = 0; \quad \nabla \cdot \vec{E} = 0; \quad \vec{j} = \gamma_e \vec{E} + \gamma_c \left( \hat{\vec{u}} \times \vec{B} \right); \]  

\[ \frac{1}{2} \, \text{tr} \left( \left( \vec{\tau} \vec{F} \right)^2 \right) - \sigma_1 = 0, \]

where \( \sigma_1 \) is the stress intensity according to von Mises; \( \vec{T} \) is the unit tensor; \( \sigma_0 \) is the spherical stress tensor; \( \vec{\tau} \) is the stress deviator; \( \sigma^p \) is the plasticity strain tensor; \( \sigma^p \) is the intensity of plastic deformations; \( E \) is the modulus of elasticity; \( \nu \) is the Poisson's ratio; \( \sigma_1 \) is the yield strength; \( E_0 \) is the modulus of hardening.

Analysis of elastic-plastic deformation of systems of electrically conductive bodies of complex geometry requires the use of numerical methods. The most universal methods include the finite element method (FEM). FEM allows considering the distribution of vector components of the electromagnetic field (EM-field) and tensor components of the deformation process within the framework of a single calculation scheme. In this case, the specific implementation of the FEM is based on the variational principle of stationarity of the total energy of the system of conductive bodies. First, to reduce the number of equations that describe the EM-field distribution, we consider the vector magnetic potential, which is related to the main vector components of the EM-field as follows:

\[ \vec{B} = \nabla \times \vec{A}; \]  

\[ \nabla \times \left( \nabla \times \vec{A} \right) = \mu_e \vec{j}, \]
where \( \vec{A} \) is the vector magnetic potential. Equations (6) are written for the case of neglecting the non-linear "magnetic" behavior of the material (which is characteristic of non-magnetic steels and alloys based on aluminum or copper) and without taking into account the "electric" component of the EM-field, because its contribution is insignificant for the analysis of the deformation of conductive bodies. The expression of total energy has the following form:

\[
E = U + W,
\]

where \( U \) is the energy of quasi-elastic deformation; \( W \) is the EM-field energy (neglecting the electric component).

\[
U = \frac{1}{2} \int_V \varepsilon : \varepsilon \cdot \varepsilon dV - \int_V \left( \vec{j} \times \vec{B} \right) \cdot \vec{u} dV - \int_{\partial V} \left( \vec{p} + \frac{1}{2} \vec{i} \times \vec{B} \right) \cdot \vec{u} dA; \tag{8}
\]

\[
W = \int_V \left( \frac{1}{2} \vec{v} \times \vec{A} \right) dV, \tag{9}
\]

where \( \varepsilon \) is the strain tensor; \((4)\,\varepsilon\) is the tensor of elasticity coefficients; \( \vec{u} \) is the displacement vector; \( \vec{p} \) is the surface load; \( \vec{i} \) is the surface current density.

The solution must satisfy the following variational equation:

\[
\delta E = \delta U + \delta W = 0. \tag{10}
\]

Equality (5) ultimately leads to the solution of two independent systems of algebraic equations with respect to nodal displacements and nodal values of the vector magnetic potential. In the case of plastic deformation, an iterative process should be considered, the features of which are discussed in the article [8].

**Calculation example.** We will apply this method to analyze the deformation of the system of electrically conductive bodies under the conditions of the MPPM technological operation. Let's consider a technological operation aimed at attraction of thin non-magnetic metal blanks, the practical direction of this operation is the correction of defects in body elements of transport equipment.

Works [5,6] show that an inductor with an auxiliary screen can be used to attract workpieces made of non-ferromagnetic materials. Figure 1 shows a model version of a single-turn inductor with an auxiliary screen.

![Fig. 1. Single-turn inductor with an auxiliary screen. 1 – current conductor of the inductor, 2 – insulation of the current conductor, 3 – auxiliary screen](image-url)
Let's analyze the elastic-plastic deformation of the inductor with the work-piece within the framework of the axisymmetric calculation scheme shown in Fig. 2. The calculation scheme contains, in addition to the elements of the technological system and the workpiece, the surrounding environment (air). The gap considered between the inductor and the workpiece should simulate the presence of a dent (defect) on the workpiece. The geometric parameters of the system are as follows: \( R_1 = 150 \text{ mm}, \ R_2 = 167 \text{ mm}, \ R_3 = 175 \text{ mm}, \ R_4 = 200 \text{ mm}, \ h_1 = 10 \text{ mm}, \ h_2 = 15 \text{ mm}, \ h_3 = 1 \text{ mm}, \ d = 1 \text{ mm}, \) insulation thickness of the current conductor – 1 mm. The dimensions of the surrounding medium were varied in order to achieve the conditions for the attenuation of the EM-field components in fields far from the source. In this case, it turned out that to reduce the EM-field components by a factor of 5, it is enough to choose the dimensions of the surrounding environment equal to twice the thickness of the inductor \( h_2. \) The source of EM-field was a current that was uniformly distributed over the cross-section of the conductor, the current density changed according to the law over time: \( j(t) = I_0 e^{-\delta t} \sin(2\pi f t), \) where the amplitude of the current \( I_0 = 40 \text{ kA}, \) frequency \( f = 2 \text{ kHz}, \) relative attenuation coefficient \( \delta = 0.3. \)

The finite-element model was created using a four-node finite element with a bilinear approximation of displacements and a circular component of the vector magnetic potential [9,10]. The using of a finite element of this type allows you to automatically satisfy the conditions at the interfaces of media with different electro-physical properties.

The solution was carried out for zero initial conditions, the boundary conditions reflected the attenuation of the EM-field at a distance from the source, as well as the fixing of the ends of the inductor and the workpiece:

\[
A|_{r_1} = 0; \quad u_r|_{r_2} = 0; \quad u_z|_{r_3} = 0. \tag{11}
\]

At the first stage of the solution, the spatio-temporal distributions of the vector components of the EM-field and the components of the electromagnetic force along the top of the workpiece were obtained. The maximum values of the normal component of the electromagnetic force are observed in the vicinity of the inductor coil.

Fig. 2. Calculation scheme of the inductor together with the workpiece. 1 – current conductor of the inductor, 2 – auxiliary screen, 3 – workpiece, 4 – insulation of the current conductor
Table 1

<table>
<thead>
<tr>
<th></th>
<th>current conductor, copper</th>
<th>auxiliary screen, steel</th>
<th>workpiece, steel</th>
<th>insulation, kaprolon</th>
<th>air</th>
</tr>
</thead>
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<tr>
<td>$\mu_1$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma_1$, (Ωm)$^{-1}$</td>
<td>$7 \times 10^7$</td>
<td>$0.2 \times 10^7$</td>
<td>$0.2 \times 10^7$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$E$, GPa</td>
<td>180</td>
<td>215</td>
<td>200</td>
<td>2.5</td>
<td>–</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.33</td>
<td>0.27</td>
<td>0.29</td>
<td>0.3</td>
<td>–</td>
</tr>
<tr>
<td>$\sigma_\tau$, MPa</td>
<td>200</td>
<td>270</td>
<td>220</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\sigma_B$, MPa</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>70</td>
<td>–</td>
</tr>
<tr>
<td>$\sigma_B$, MPa</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>90</td>
<td>–</td>
</tr>
</tbody>
</table>

Next, we consider the combined deformation of the inductor and the workpiece, the results are given for the time maximum. In fig. 3 shows the deformed state of the system, it can be seen that the displacements of the workpiece significantly exceed the displacements of the inductor. The maximum values of workpiece motions are observed around its center. In fig. 4 shows the distribution of stress intensity in the inductor and the workpiece. The maximum stress intensity values in the workpiece are observed in the vicinity of the end zone, which is determined by the fixing...
conditions specified there. The "second" maximum is observed in the center, which is due to the maximum displacements in this zone. Note that the maximum values of stress intensity in the workpiece do not reach the yield point, that is, irreversible deformation under these conditions is not achieved. In the inductor, the maximum intensity is observed in the vicinity of the turn, and its values almost reach the yield point of copper and the tensile strength limit for kaprolon.

Thus, under the considered conditions, the goal of the technological operation is not achieved. When the magnitude of the current strength increases, the magnitude of the stresses will also increase, while the intensity of the stresses will reach vaporous values first in the inductor. To achieve a complete technological operation, you can use a multi-turn version of the inductor.

**Conclusions:** the article formulates an actual scientific and practical problem of elastic-plastic deformation of structural elements under the action of an electromagnetic field, presents a mathematical formulation of the problem, and gives an example of a solution.

**ЛІТЕРАТУРА**

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