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MATHEMATICAL MODEL OF TRANSIENT MODES OF ELECTRIC SHIPS' PROPULSION COMPLEXES WITH THRUSTERS

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Summary

The purpose of the work *is to develop a mathematical model and a method for calculating the maneuvering modes of propulsion electrical power plants with thrusters.* ***Methodology****. During maneuvering, both the propulsion electric power plant, the thrusters, and the ship's hull operate in transient modes. The durations of transient processes in them are commensurate, so the analysis of the behavior of any constituent part of the propulsion complex should be carried out in unity with all other parts.* ***The results.*** *A mathematical model and a method for calculating transient modes of propulsion complexes of electric ships with thrusters have been developed. The operation of thruster propellers is considered taking into account the hydrodynamic processes of their interaction with the ship's hull, viscous losses and viscous interaction. The criteria of dynamic similarity and the generalized dimensionless parameters of the complex affecting the maneuverability indicators were revealed. The workability of the model and the calculation method is confirmed by the results of numerical simulations, which illustrate the effect of the operation of the thrusters on the electric ships' maneuverability.* ***Scientific novelty.*** *The model and the calculation method allow calculating the main mode indicators and indicators of the quality of maneuvering of modern electric ships equipped with thrusters. At the same time, control of operating parameters of all components of the propulsion electric power plant is carried out. There is an opportunity to design propulsion plants according to the final result – according to the quality of the vessel's maneuverability.* ***Practical value.*** *The calculation method will make it possible to conduct a study of the behavior of propulsive complexes during maneuvering, to optimize the control* *process, and to find ways to increase the electric ships' maneuverability. Bible. 15, tab. 2, Fig. 7.*

Key words: *electric ship's propulsive complex, thrusters, mathematical model of transient modes.*

МАТЕМАТИЧНА МОДЕЛЬ ПЕРЕХІДНИХ РЕЖИМІВ ПРОПУЛЬСИВНИХ КОМПЛЕКСІВ ЕЛЕКТРОХОДІВ З ПІДРУЛЮЮЧИМИ ПРИСТРОЯМИ

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Анотація

Метою роботи *є розробка математичної моделі та методу розрахунку перехідних режимів гребних електроенергетичних установок з підрулюючими пристроями.* ***Методика.*** *При маневруванні і гребна електроенергетична установка, і підрулюючи пристрої, і корпус судна працюють на перехідних режимах. Тривалості перехідних процесів у них співмірні, тому аналіз поведінки будь-якої складової частини пропульсивного комплексу повинен проводитися в єдності з усіма іншими частинами.* ***Результати.*** *Розроблено математичну модель та метод розрахунку перехідних режимів пропульсивних комплексів електроходів з підрулюючими пристроями. Робота гребних гвинтів підрулюючих пристроїв розглянута з урахуванням гідродинамічних процесів їх взаємодії з корпусом судна, в'язкісних втрат і в'язкісної взаємодії. Виявлено критерії динамічної подібності та узагальнені безрозмірні параметри комплексу, що впливають на маневрені показники. Працездатність моделі та методу розрахунку підтверджені результатами чисельного моделювання, що ілюструють вплив роботи підрулюючих пристроїв на маневреність електроходів.* ***Наукова новизна****. Модель і метод розрахунку дозволяють розраховувати основні режимні показники та показники якості маневрування сучасних електроходів, обладнаних підрулюючими пристроями. Одночасно здійснюється контроль режимних показників всіх складових частин гребної електроенергетичної установки. З'являється можливість проектування гребних установок за кінцевим результатом – за якістю маневрування судна.* ***Практичне значення.*** *Метод розрахунку дозволить проводити дослідження* *поведінки пропульсивних комплексів при маневруванні, оптимізувати процес управління, відшукувати шляхи підвищення маневреності електроходів. Бібл. 15, табл. 2, рис. 7.*

Ключові слова: *пропульсивний комплекс електрохода, підрулюючі пристрої, математична модель перехідних режимів.*

Relevance. The controllability of the vessel, that is, its ability to follow a given trajectory or change the direction of movement at the command of the shipmaster, is one of the important seafaring qualities that determine the operational capabilities of the ship and, to a large extent, the safety of navigation. Traditionally, the main means of controlling the ship is the stern rudder. However, the action of the rudder is effective only if there is a significant speed of the ship. In certain operating conditions (on a limited fairway, on moorings, when maneuvering in limited conditions, etc.), it is impossible to develop a speed sufficient for effective control of the vessel. Poor controllability of ships in similar conditions was the cause of most accidents: collisions; groundings and groundings on rocks; impacts against the walls of piers, etc. To ensure controllability at low speed and without speed, thrusters (*Th*) are installed on ships, which create additional steering thrust. They can create lateral force at any, however small, speed of the vessel. After 1965, when Kawasaki launched *Th*s production, they began to be used as the most important elements of the propulsion system and began to be widely used for almost any type of vessel, from dry cargo, passenger, container carriers to offshore. The power of the thruster electric motors varies from several dozen to 5000 kW. It should be enough to ensure the required thrust value of the thruster propellers. The magnitude of this power depends on many factors, and first of all on the required maneuverability properties, and even for vessels with a moderate degree of maneuverability. The share of power (K_{TR} – coefficient) of *Th* electric motors relative to the power of the propulsion electrical power plant (PEPP) reaches K_{TR} = 0,24 (see table 1).

Table 1

Proportion of engine power of propulsion devices in PEPP

Name of the vessel	**Arctikaborg**	**Europa 2 (Hapaq-Lloyd)**	**Vasily Dinkov**	**Mikhail Ulyanov**	**Carnival Miracle**
Type	Icebreaker	Cruise vessel	Arctic tanker	Arctic tanker	Cruise vessel
Vessel length, m	65,1	225,4	257,3	257,7	292,5
Th total power, kW	150	1750	2000	4000 (2×2000)	5730 (3×1910)
PEPP total power, kW	3240 (2×1620)	14500 (2×7250)	20000 (2×10000)	17000 (2×8500)	35200 (2×17600)
Power share K_{TR} of *Th* motors	0,05	0,12	0,1	0,24	0,16

Thus, the thruster is one of the most powerful electricity consumers on the ship. Its work has a very significant effect on the operation of the ship's power plant, on the electrical power quality of the ship network. Therefore, a number of tasks arise in the process of designing and operating electric motors, the most important of which is to achieve the necessary maneuverability of the vessel, while ensuring the proper operation of both the ship's electrical power sources and the ship's general consumers. Their solution can be sought by modeling the maneuvering modes of operation of electric ships with thrusters.

State of the issue. The operation of thrusters during ship maneuvering has been considered in numerous studies. But for the most part, they are aimed at solving specific applied tasks that relate specifically to thrusters. In particular:

Works [1–4] present the results of experimental and numerical research of hydrodynamic characteristics and the effect on the efficiency of bow thrusters (*ThB*): different chamfer depths and tunnel opening angles [1]; different angles of water inflow and fillet radii [2]; interaction of several bow thrusters at "zero" ship speed [3], pressure fluctuations in the tunnel [4].

Work [5] is devoted to the modeling of the load on the *ThB* in operating conditions. The analysis of the operation of the *ThB* motor in conditions of voltage imbalance that occurs on the ship was carried out in [6]. Issues of operational control to increase the energy efficiency of ship micro nets (including *ThB*) are considered in work [7].

Article [8] proposes a method of building the trajectory of a ship's turn using a control device. The expert system that ensures the design of the ship's power subsystems, in particular the *Th* subsystem, is presented in work [9]. The simplest mathematical models are used here for the approximate selection of structural elements of the thrusters. The proposed expert system is intended to check at the initial stages of design the compliance of the analyzed structure with the requirements of classification societies.

A number of works [10; 11] consider the issues of transition from the most common independent control bodies for thrusters, propellers and ship rudders to an integrated, power-optimized, generalized control system using a joystick with three degrees of freedom. In particular, [10] investigates the problems of thrust distribution in the propulsion electrical power plant, consisting of propulsion motors (*PM*), tunnel or azimuth thrusters during their interaction. The purpose of the study is to increase the efficiency (reduction of energy consumption while maintaining maneuverability) of movement and the accuracy of thrust generation from the motion controller. In [11], the results of studies of potential energy savings in the operation of a RoPax twin-screw ferry with a bow thruster are presented. The research was carried out on the basis of a simulator and is aimed at improving maneuvering in sea ferry ports, that is in marine areas sensitive to harmful emissions into the environment.

Analysis of the published results of earlier studies allows coming to the following conclusions.

The main task that researchers set themselves is to assess the maneuverability of vessels equipped with thrusters. At the same time, only the tasks of controlling the movement of the vessel are solved. The authors try not to touch the propulsion plant, to avoid issues of interaction between the propellers and the ship's hull. Most often, the works deal with the issues of improving the structural parameters of the *Th*, creating the software of the simulators, which should be used to practice the techniques of controlling the vessel movement.

Ship electrical power equipment is not considered. It is assumed that propulsion motors and propellers work in stable mode, at low speed. Thus, from the single ship propulsion complex (ship hull, propellers, propulsion electrical power plant) are, in fact, separated and only the ship hull is considered.

At the same time, a number of questions arise. And is the propulsive complex with thrusters capable of performing the planned maneuver? Is the ship's electrical power plant able to ensure the operation of this complex? What will be the load on the electric power system? Will the protection system not work in this case and will the maneuvering process face a dangerous situation of failure to maneuver in a limited water area?

To answer these questions, a systematic approach is necessary: the electric ship's propulsion electrical power plant with thrusters must be considered together with all the components of a single ship propulsion complex. A suitable mathematical apparatus is necessary in order to solve the problems of controllability of vessels with thrusters during maneuvering operations, for the correct layout and design of the electrical power system capable of providing the vessel with high maneuverability qualities, for the possibility of finding the best options for controlling such a complex propulsive system in various modes of operation. The purpose of this work is to build a mathematical model of transient modes of electric ships' propulsive complexes with thrusters.

Problem solving method. As a basis for the analysis of the maneuverability characteristics of the electric ships' propulsion complexes with *Th* it is proposed to lay a mathematical model of the transient modes of the propulsion complex with a single shipboard electric power plant, which is presented in works [12; 13]. The version of the propulsion plant given by them corresponds to the classic layout of the electric propulsion system based on frequency-controlled propulsion motors. The electrical power plant includes heat engines, synchronous generators, ship-wide power consumers and several (usually two or three) "power" propulsive circuits. Each circuit includes a frequency converter, an asynchronous propulsion motor and a propeller. In addition to the propulsion electric power plant, the propulsive complex includes the rudder and the ship's hull.

The model and the calculation method developed on its basis allow calculating the current values of the main mode indicators of all constituent parts of the propulsive complex during maneuvers:

– for each power circuit: angular speed of rotation, torque and the heat engine power; generator voltage and current; voltage, current, torque and angular speed of rotation of the propulsion motor;

– according to the parameters of the ship's movement: the speed of movement in the coordinate system connected and not connected to the ship; angular speed of rotation around the vertical axis; drift angle and vessel course angle.

If necessary, any other regime indicators obtained in the process of calculations can be registered.

The presented model has shown its efficiency. By analogy with it, and in accordance with the goal set in this work, it is proposed to develop and introduce into the propulsive complex a mathematical description of the processes that take place in the thrusters during ship maneuvering. This will make it possible to analyze the maneuverability characteristics of modern electric ships, evaluate the operating modes of their electric power plants, and open up opportunities for optimal design and optimal control of ship propulsion complexes.

The thruster propeller, like any ship's propeller, interacts with the environment, with the walls of the channel, with parts of the *Th* itself. Despite the fact that the main mode of the thruster operation is to control the vessel in the absence of its motion, at low speed it, nevertheless, significantly affects the vessel movement and is used during maneuvering. The operation of a real thruster is affected by viscous losses and viscous interaction, channel configuration, and much more [14]. Based on this, taking into account the difficulties in describing the working processes of propellers of propulsion and steering systems, the inaccuracy in determining the main coefficients of interaction between the propellers and the ship's hull, researchers try to avoid considering the hydrodynamic processes of the interaction of the thruster propellers with the ship's hull. They try to use some generalized coefficients, the accuracy of which is small, and the physical value is not completely defined. Such a simplified approach is acceptable to a certain extent if it is only about an approximate assessment of the maneuvering characteristics of the vessel. If, in the course of research, questions are asked from the perspective that was determined in the form of the goal of this work, if it is about the ability of the electric vessel and its electric power plant to perform the task while maintaining the operability of all ship systems, it is mandatory to take into account the hydrodynamic processes of the interaction of the thruster propellers with the ship's hull. It is also obligatory to take into account the interaction of the remaining components of the ship's electrical power plant.

It is proposed to lay the well-known Hoffmann method [14] as the basis for the hydrodynamic calculation of the thruster propellers. The method is based on serial test diagrams of propellers in a thin cylindrical pipe. K4-55 type propellers were used (B4-55 Wageningen basin propellers with a blade contour in the form of a Kaplan turbine blade).

The following should be noted immediately. Naturally, for the propellers with other profiles and parameters, the diagrams will differ. But firstly, it does not affect the method of calculating hydrodynamic characteristics. Secondly, the system of dimensionless parameters, which is embedded in the mathematical model developed in this work, and the adopted approach to the analysis of transient modes of operation (in which the generalized dimensionless parameters of the electric ship's propulsive complex are used as varying parameters) completely eliminate the difference between these parameters.

The calculation is based on the following provisions.

1. During the operation of the propeller in a thin cylindrical pipe in the mooring mode (vessel speed $v = 0$, propeller advance $\lambda = v/Dn$, where D and n are the diameter of the propeller and its rotation frequency), the relative advance of the propeller according to the speed of water flow $\lambda_s = v_s/Dn$ in the pipe depends only on the pitch ratio of the propeller P/D (Fig. 1).

The propeller thrust in this mode (neglecting the small resistance of the thin pipe $\xi \approx 0$) corresponds to

$$P_{Th} = \rho A_0(\omega^2 / 2) = \rho A_0(v_s^2 / 2), \quad (1)$$

where ρ – water specific gravity; A_0 – area of the propeller disk; ω – axial component of speed caused by the propeller; v_s – speed of water flow through the propeller.

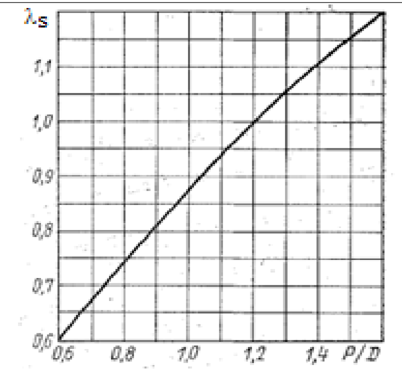

Fig. 1. Dependence of the relative advance of the propeller on the step ratio

The load factor of the propeller according to the speed of the water inflow

$$\sigma_{Ps} = 2T / \rho v_s^2 A_0 = 1{,}0. \quad (2)$$

The thrust coefficient $\sigma_{Ps} = 2T / \rho v_s^2 A_0 = 1{,}0$.

The propeller thrust in this mode (neglecting the small resistance of the thin pipe $\xi \approx 0$) corresponds to

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2. In running mode, the thrust, the propeller load factor and the thrust factor are determined by ratios

$$P_{Th} = \rho A_0 \left(v + \frac{\omega}{2} \right) \frac{\omega}{2} = \frac{\rho A_0}{2} \left(v_s^2 - v^2 \right); \quad (3)$$

$$\sigma_{Ps} = \frac{2T}{\rho v_s^2 A_0} = 1{,}0 - \left(\frac{v}{v_s} \right)^2; \quad (4)$$

$$k_P = \left(\frac{\pi}{8} \right) \left(\lambda_s^2 - \lambda^2 \right). \quad (5)$$

3. During the operation of the propeller in a cylindrical pipe, which has resistance $\xi_K \neq 0$ in the mooring mode, the additional thrust required to overcome this resistance and the additional part of the thrust coefficient are equal to

$$\Delta P_{Th} = \zeta_k \rho A_0 \left(\frac{v_s^2}{2} \right); \quad (6)$$

$$\Delta k_P = \left(\frac{\pi}{8} \right) \zeta_k \lambda_s^2 . \quad (7)$$

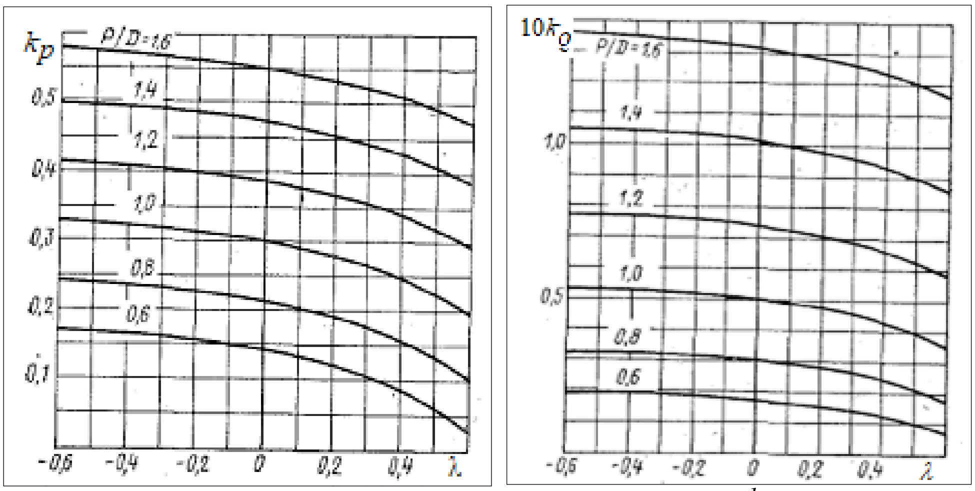
The thrust coefficient

$$k_P = \left(\frac{\pi}{8} \right) \lambda_s^2 \left(1 + \zeta_k \right) \quad (8)$$

Thus, it follows from ratios (5) and (8) that the operation of the propeller in a cylindrical pipe with a resistance coefficient $\xi_K \neq 0$ is equivalent to the operation of the propeller-cylindrical pipe system with a negative relative advance of the propeller

$$\lambda = -\lambda_s \sqrt{\zeta_k} < 0 . \quad (9)$$

Paper [14] presents diagrams of serial tests of propellers in a cylindrical pipe, extrapolated to the region of small negative values of the negative stroke. These are graphical dependences of the thrust coefficient k_P and the coefficient of the moment of resistance k_Q of the propeller as a function of the relative advance and the step ratio *P/D* (Fig. 2). The diagrams can be successfully used to calculate in the first approximation the main characteristics of the thruster propellers during the design and calculation of their main operating parameters at stable modes.



a *b*

Figure 2 – Dependencies:
a – of the thrust coefficient k_P; b – the resistance moment coefficient k_Q
as a function of the relative advance and step ratio

However, the graphical method presented in [14] does not allow calculating the operation of the *Th* in maneuvering modes, since both the step ratio and the advance of the propellers are continuously changing. It is necessary to build a method for calculating the current values of the main operating parameters of the propellers (thrust and moment of resistance on the shaft) with time-varying step ratio (which is a control parameter) and propeller advance, and to integrate the obtained algorithm into the mathematical model of a single ship propulsion complex.

Analytical dependencies play an important role in the method of calculating the main regime indicators:

a) the relative advance of the propeller according to the water flow rate λ_s from *P/D* (Fig. 1). This dependence can be represented by a linear equation

$$\lambda_s = 0{,}3015\ P/D; \qquad (10)$$

b) dependences of the thrust coefficient k_P and the coefficient of the resistance moment k_Q of the propeller on the advance of the propeller λ and the step ratio P/D

$$k_P = 0{,}0314(P/D)^2 + 0{,}3417P/D - 0{,}0706 + 0{,}05\lambda, \qquad (11)$$

$$k_Q = 0{,}0434(P/D)^2 + 0{,}025P/D - 0{,}0179 + 0{,}005\lambda. \qquad (12)$$

When calculating the maneuvering modes, the thrust P_{Th} and the resistance moment M_{Th} of the thruster propeller are found by the method of successive approximations at each step of the integration of the system of equations, which describes the transient processes in the electric ship's propulsive complex.

The operation of the propeller begins after the output of the drive electric motor to the steady operation mode.

At the initial instant of time, the propeller advance λ, the pitch ratio *P/D*, and the resistance moment coefficient k_Q are zero. The moment of resistance on the electric motor shaft is determined by the moment of resistance of frictional forces. As the step ratio (control signal) increases, the coefficient k_Q begins to increase, correspondingly, the resistance moment of the thruster propeller increases, which can be determined with a sufficient degree of accuracy from the ratio

$$M_{Th} = \frac{9550\left(k_Q 2\pi D^5 n^3\right)}{n}. \quad (13)$$

The torque M_{MTh} on the motor shaft M_{Th} increases. The electric motor, maintaining (in accordance with the statics of the working section of the mechanical characteristics) revolutions, "takes" the load. With an increase in the step ratio, the advance of the thruster propeller λ_s appears, which will initially be negative. This is the "viscous" component of advance. It is determined from the obtained analytical dependence (10) λ_s on *P*/*D*. After that, the specified value of the advance of the propeller is found

$$\lambda = -\lambda_s \sqrt{\zeta_k}. \quad (14)$$

For the refined value of λ, new values of the coefficient k_Q and the moment M_{Th} are calculated (12).

According to the obtained dependences (11) of the thrust coefficient k_P on the advance λ and the step ratio *P*/*D*, the thrust coefficient, the propeller thrust P_{Th} and the thruster thrust T_{Th} are calculated

$$P_{Th} = k_T \rho D^4 n^2; \quad (15)$$

$$T_{Th} = P_{Th}\,(1 + t), \quad (16)$$

where t – the absorption coefficient.

During the operation of the *Th* in the running mode of the vessel, the jet of water flowing out of the channel at the speed vs interacts with the water flow flowing into the jet at the speed of the vessel v at an angle of 90°. As a result of the mixing of flows, complex turbulence processes arise, which lead to the distortion of the jet. The degree of development of these processes depends on the ratio between v and v_s – on the parameter $m = v/v_s$.

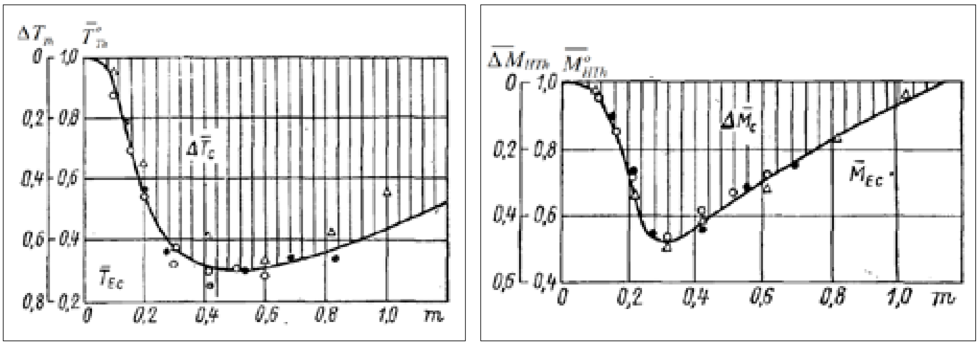
The interaction of the jet with the water flow carried by this jet leads to the formation of a discharge zone between the ship's hull and the jet. As a result, the suction force ΔT_{Th} appears, which is directed to the side, opposite to the action of the propeller thrust T_{Th}, and is applied in the area between the *Th* and the center of the ship.

As a result, the total force acting on the ship and its moment relative to the gravity center decrease. According to the results of the test in the experimental basin, the dependencies [14] for the relative value of the resulting transverse force $\overline{T_{Th}^0}$ and the relative resulting moment $\overline{M_{HTh}^0}$ (Fig. 3) were obtained.

$$\overline{T_{Th}^0} = \frac{T_{Th} - \Delta T_{Th}}{T_{Th}}; \quad (17)$$

$$\overline{M^0_{HTh}} = \frac{T_{Th}h_h - \Delta T_{Th}h_c}{T_{Th}h_h} = \frac{T_{Th}h_h - \Delta T_{Th}h_c}{T_{Th}\overline{h_h}}, \qquad (18)$$

where h_h – coordinate of application of propeller thrust T_{Th} and its relative value $\overline{h_h} = \frac{h_h}{L}$; h_c– is the coordinate of application of the suction force ΔT_{Th} and its relative value $\overline{h_c} = \frac{h_c}{L}$.



a b

Fig. 3. The relative value of the:
a – resulting transverse force; b – moment acting on the ship during the operation of the ThB with the simultaneous translational movement of the ship

$$\overline{T^0_{Th}} = \frac{T_{Th} - \Delta T_{Th}}{T_{Th}}; \qquad (17)$$

$$\overline{M^0_{HTh}} = \frac{T_{Th}h_h - \Delta T_{Th}h_c}{T_{Th}h_h} = \frac{T_{Th}h_h - \Delta T_{Th}h_c}{T_{Th}\overline{h_h}}, \qquad (18)$$

where h_h – coordinate of application of propeller thrust T_{Th} and its relative value $\overline{h_h} = \frac{h_h}{L}$; h_c– is the coordinate of application of the suction force ΔT_{Th} and its relative value $\overline{h_c} = \frac{h_c}{L}$.

The dependence of the position of the application point of the suction force on the parameter *m* is shown in Fig. 4.

As can be seen from Fig. 3, the negative effect of the suction force is manifested at low speeds of movement – from 3 to 5 knots. At the same time, the thrust of the thruster propellers is at the level of 30% – 40%, and the moment from this force does not exceed 50% of the corresponding values when the vessel is not moving. This is a very

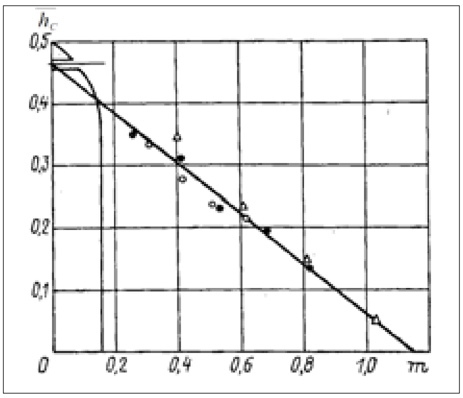


Fig. 4. The position of the suction force application point ΔT_{Th} depending on the parameter m

dangerous situation, since it is precisely at such speeds that thrusters are used. Therefore, taking into account the suction force during the operation of the thrusters at low speed is mandatory.

To obtain analytical dependences $\overline{T^0_{Th}}$ and $\overline{M^0_{HTh}}$ on the parameter *m*, we will use the MATLAB function of data approximation by polynomials.

According to its results, the following dependencies were obtained:

$$\overline{T^0_{Th}} = -1{,}7191m^4 + 1.6259m^3 + 2.3692m^2 - 2{,}8864m + 1{,}0373; \quad (19)$$

$$\overline{M^0_{HTh}} = -4{,}1084m^4 + 6{,}0548m^3 - 0{,}3785m^2 - 1{,}699m + 1{,}0403. \quad (20)$$

The dependence on the parameter *m* is linear

$$\overline{h_c} = 0{,}461 - 0{,}408m. \quad (21)$$

The developed model of the hydrodynamic calculation of the main mode parameters of the thruster propellers is integrated into the generalized mathematical model of the transient modes of the electric ship's propulsion complex. [15]. Taking into account the thruster, the structural scheme of the complex will undergo significant changes. In this case, it has the form shown in Fig. 5.

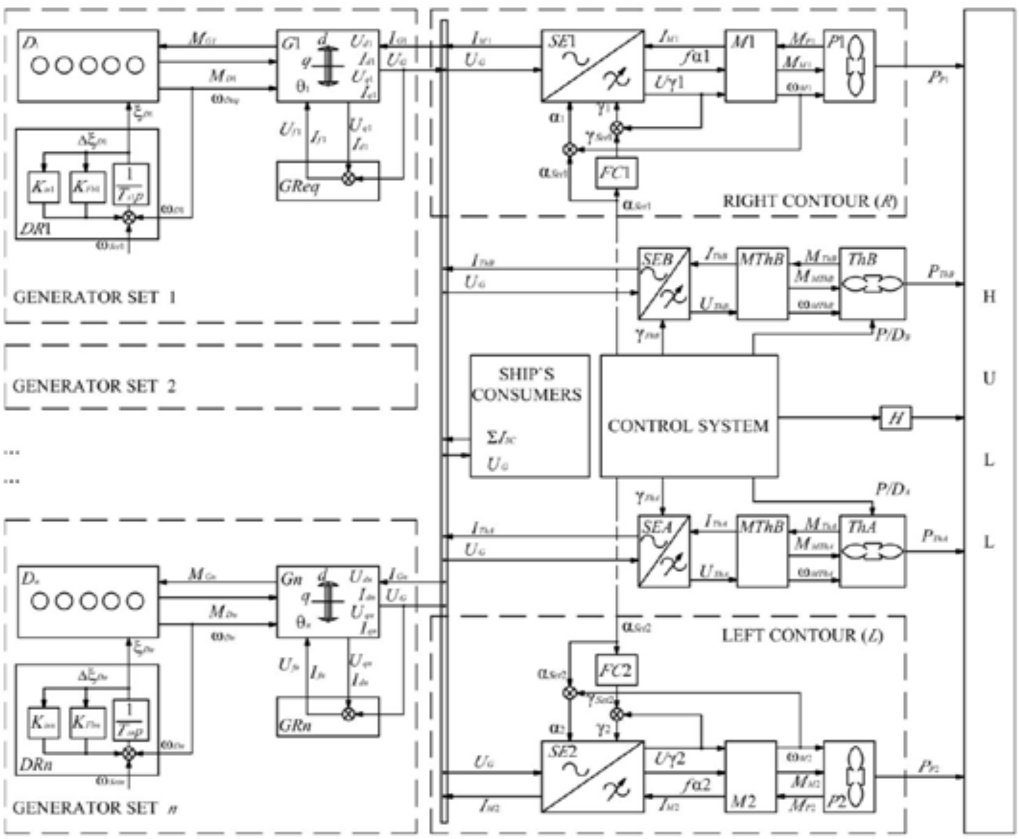


Fig. 5. Structural diagram of the electric ship's propulsive complex with thrusters

The propulsive complex includes the following main components:

– electric generation system from several generator units with active and reactive power distribution systems: $D1, D2, \ldots, D_n$ – heat engines; $G1, G2, \ldots, G_n$ – synchronous generators (SG);

– two "power" circuits, in each of which: *SE*1, *SE*2 – frequency converters; *M*1, *M*2 – asynchronous propelling electric motors; *P*1, *P*2 – propellers;

– bow and stern thrusters, which include: *SEB* and *SEA* – voltage converters (for starting) bow and stern thrusters; *MThB*, *MThA* – drive electric motors; *ThB* and *ThA* – bow and stern thruster propellers;

– rudder – *R*;

– ship's hull.

Elements of the automatic regulation system and the main parameters connecting the power units and control signals: primary speed regulator – *DR*; automatic generator voltage regulator – *GR*; M_D and ω_D – torque and angular speed of rotation of the heat engine; M_G – electromagnetic moment of the generator; U_d and U_q – voltages of the generator along the longitudinal and transverse axes (internal coordinates); I_d and I_q – generator currents along the longitudinal and transverse axes (internal coordinates); U_G – generator output voltage; ω_{Set} – setting of the angular speed of rotation of the speed controller; ξ_D – stroke of the fuel pump rail; $1/T_s p$ – servo motor link; K_{Fb} and K_{is} – amplification coefficients of rigid and flexible (isodromic) feedback links; U_f and I_f are voltage and excitation current of the synchronous generator; I_G, I_M – *SG* and *PM* currents; α_{Set} and γ_{Set} – relative frequency and voltage of the converter (control signals); α and γ – relative frequency and voltage at the output of the converter (taking into account feedback); *FC* – a functional converter that forms the law of frequency control $\gamma = f(\alpha)$; M_P and P_P – moment and thrust of propellers; M_M and ω_M – torque and angular speed of rotation of the electric motor; U_{Th} and I_{Th} – voltage and current of thruster electric motors; M_{Th} and ω_{Th} – moment and angular speed of rotation of electric motors *Th*, P_{Th} and M_{Th} – thrust and moment of resistance of thruster propellers; γ_{Th} – voltage control signals of the thruster electric motor; *P*/*D* is the pitch ratio of thruster propellers.

The main ratios of the energy transmission channel to propulsion electric motors (abbreviated version of the generalized mathematical model of transient modes) are presented below. The system of equations describing the transient modes of operation of the propulsive complex is presented in relative units. Relative values of mode indicators are indicated by symbols with a dash on top. (The index "0" corresponds to the values of these indicators, when the electric motor is operating in a stable mode with the nominal power of the PEPP).

Equations of motion of generator units of a ship's power plant with regulators of their rotation speed

$$\frac{d\overline{\omega_D}}{dT} = N_D\left(\overline{M_D} - \overline{M_G}\right), \tag{22}$$

where $N_D = \dfrac{M_{D0}L}{J_D\omega_{D0}\nu_0}$ – criterion of dynamic similarity; J_D – the moment of inertia of the engine and generator reduced to the shaft of the heat engine; T – relative (dimensionless) time $T = \dfrac{\nu_0}{L}t$; v_0 – vessel speed; L – vessel length; t – the time.

The torque of the heat engine is represented as the relative movement of the fuel pump rail

$$\overline{M_D} = \overline{\xi_D}\,. \tag{23}$$

Electromagnetic resistance moment of the generator

$$\overline{M_G} = -K_{G1}\overline{I_G^2}\sin\psi_G\cos\psi_G + K_{G2}\overline{I_f}\,\overline{I_G}\cos\psi_G\,,$$

where ψ_G – the angle of phase shift between vectors $\overline{E_G}$ and $\overline{I_G}$; K_{G1}, K_{G2} – dimensionless parameters of the propulsive complex.

The mathematical model of transient modes of synchronous generators is obtained taking into account the Park-Horev equations. At the same time, a simplified version was used, which provides accuracy comparable to the accuracy of the description of transient processes of other constituent parts of the complex. The voltage at the output of the generator is determined by the ratio

$$\overline{U_G} = \sqrt{\left(\overline{U_d^2} + \overline{U_q^2}\right)}\,, \tag{24}$$

where $\overline{U_d}$ and $\overline{U_q}$ are generator voltages according to internal *d–q* coordinates.

Frequency converters (for frequency control of propelling electric motors) and voltage converters (for smooth starting of thruster electric motors) are represented by inertialess electricity converters. The output voltage of converters U_M (voltage at the input of electric motors $U_{\gamma 1}$, $U_{\gamma 2}$, U_{ThB}, U_{ThA}) depends on the input voltage (voltage of the generator U_G) according to the ratio

$$\overline{U_M} = \gamma\overline{U_G}\,, \tag{25}$$

where γ – a control parameter.

Equations of motion of frequency-controlled PEMs

$$\frac{d\overline{\omega_M}}{dT} = N_M\left(\overline{M_M} - \overline{M_P}\right), \tag{26}$$

where

$$N_M = \frac{M_{M0}L}{J_M\omega_{M0}\nu_0} \tag{27}$$

criterion of dynamic similarity;

J_D – is the moment of inertia of the motor, the propeller and the mass of water attached to it, applied to the shaft of the propulsion electric motor.

A full description of the transient processes in the propulsion electric power plant, with justification of the assumptions made, as well as the method of calculating the dimensionless parameters and similarity criteria of the propulsive complex, obtained in the process of building the model, is given in [12].

The propulsion electric power plant is controlled from the control panel (CP) on the bridge. The output signals of the CP are:

– control signals on the *SE*1 and *SE*2 frequency converters of propulsive electric motors PEMs (in terms of frequency α_{Set1}, α_{Set2} and, with the help of functional converters *FC*1, *FC*2, in terms of voltage – γ_1, γ_2);

– voltage control signals γ_{ThB}, γ_{ThA} on *SEB*, *SEA* voltage converters for soft start of thruster electric motors;

– control signals according to the step ratio *P/D* of the thruster propellers *ThB* and *ThA*;

– steering angle – β_R.

The equations of the control system link the signals applied to the inputs of the control system with the voltage at the output of the corresponding control channel.

In the general case, this connection has the following form:

$$A_{Kr}(t)\left(U_{out}\right)_{Kr} = K_{Kr}^{CS}\sum_i K_i^{CS}\left(U_{Ui} - U_{C\,i}\right), \tag{28}$$

where *K* – a control object; *r* – controlled parameter; A_{Kr} – a functional dependence of control devices on time, including the derivative; $(U_{out})_{Kr}$ – a control signal at the input of the *K*-th object according to the *r*-th parameter; K_{Kr}^{CS} – amplification factor for the *r*-th control parameter of the *K*-th object; U_{Ui} – signals of control and corrective connections; U_{Ci} – comparison (cut-off) voltage; K_i^{CS} – the amplification factor for the *i*-th control signal.

In each specific case, the automatic control system has its own "set" of control signals for each control channel.

Electric ship's hull.

With the appearance in the mathematical model of the propulsive complex of thrusters, the equations of the ship motion undergo significant changes. Additional forces and moments from thrusters appear in the right-hand sides of these equations.

The components of the electric ship's speed of movement along the *X*, *Y* axes and the speed of rotation around the *Z* axis, with a modern approach to describing the movement of the vessel on the free surface of the water in the *GXYZ* coordinate system associated with it, will take the following form:

$$\frac{d\overline{\nu_X}}{dT} = C_{\lambda 2}\overline{\nu_Y}\,\overline{\Omega_Z} + N_X\left\{\sum_J K_{Pj}\overline{P_{ej}} - C_{RX}\beta_{RP}\overline{\nu^2} - \overline{R_X}\right\}; \tag{29}$$

$$\frac{d\overline{\nu_Y}}{dT} = -\frac{1}{C_{\lambda 2}}\overline{\nu_X}\,\overline{\Omega_Z} + \frac{N_X}{C_{\lambda 2}}\left\{\sum_J K_{Pj}\alpha_{jz}\overline{P_{ej}} + C_{RY}\beta_{RP}\overline{\nu^2} \pm \sum_h k_h\overline{P_{Thh}} - \overline{R_Y}\right\}; \tag{30}$$

$$\frac{d\overline{\Omega_Z}}{dT} = -\frac{N_\Omega}{N_X}C_{\lambda 21}\overline{\nu_X}\,\overline{\nu_Y} +$$

$$+N_\Omega\left\{\sum_j K_{Pj}h_{Pj}\overline{P_{ej}} + C_{RY}\overline{X_R}\beta_{RP}\overline{\nu^2} \pm \sum_h k_h\overline{P_{Thh}}\,\overline{h_h} + \left(\overline{M_{PZ}} - \overline{M_{DZ}}\right)\right\}, \tag{31}$$

where

$$\overline{R_X} = \left\{C_{11}\cos 1{,}5\beta_{dr} - C_{12}\sin^4 1{,}5\beta_{dr} + C_{13}\left(\frac{2\beta_{dr}}{\pi}\right)^3\right\}\overline{\nu^2} \tag{32}$$

longitudinal component of the water resistance of the ship's movement;

$$\overline{R_Y} = \left\{C_{21}\sin 2\beta_{dr}\cos\beta_{dr} + C_{22}\sin^2\beta_{dr} + C_{23}\sin^4 2\beta_{dr}\right\}\overline{\nu^2} \tag{33}$$

transverse component of the water resistance of the ship's motion;

$$\overline{M_{PZ}} - \overline{M_{DZ}} = \left[C_{61}\sin 2\beta_{dr} + C_{62}\sin\beta_{dr} + C_{63}\sin^3 2\beta_{dr} + C_{64}\sin^4 2\beta_{dr}\right]\overline{\nu^2} - C_{65}\overline{\Omega_Z}\,\overline{\nu^2} \tag{34}$$

positional and dynamic components of the turning resistance moment of the vessel;

$\sum_h k_h \overline{P_{Thh}}$ – total propeller thrust of the thrusters;

$\sum_h k_h \overline{P_{Thh}}\,\overline{h_h}$ – total torque from the thruster propellers, relative to the ship's center of gravity;

$N_X = \dfrac{L\sum K_{Pj}P_{ej0}}{(m+\lambda_{11})\nu_0^2}$, $N_\Omega = \dfrac{L^3\sum K_{Pj}P_{ej0}}{2(J_Z+\lambda_{66})\nu_0^2}$ – dynamic similarity criteria;

X_R – distance from the center of the coordinate system to the steering wheel; P_{ej} and K_{Pj} – useful thrust of the propeller and its share in the total thrust, respectively; L – vessel length; m – mass of the vessel; ρ – water specific density;

λ_{11} and λ_{22} – attached masses of water along the X and Y axes; J_Z – ship's moment of inertia during rotation around the Z axis; λ_{66} – attached moment of inertia of water; β_{dr} – drift angle; $\overline{P_{Thh}}$, $\overline{h_h}$ and k_h – the relative thrust, its shoulder, and the share of the thrust of the corresponding thruster propeller, at the maximum step ratio in the total thrust of the propulsive propellers.

Rudder attack angle

$$\beta_{RP} = K_R\beta_R - \chi_C\left(\operatorname{arctg}\beta_{dr} - \varepsilon\frac{\overline{\Omega_Z}}{\overline{\nu}}\right), \quad (35)$$

where χ_C – the coefficient of influence of the electric ship's hull and its propellers on the rudder; ε – the value determined by the l_R/L ratio (l_R is the distance between the rudder and the middle frame).

In the process of building the model, dimensionless parameters of the propulsive complex $C_{\lambda 2}$, $C_{\lambda 21}$, $C_{\lambda 2}$, C_{RX}, C_{RY}, C_{11}, C_{12}, C_{13}, C_{21}, C_{22}, C_{23}, C_{61}, C_{62}, C_{63}, C_{64}, C_{65} were obtained (they are calculated according to the ratio given in work [7]). It is these parameters that determine the main indicators of the electric ships' maneuverability, affect the performance of the propulsion electric power plant, the quality of the electric power of the ship's network (and, accordingly, the work of the ship's general consumers).

The transition to generalized dimensionless parameters allows approaching the solution of another important problem. Unfortunately, the processes of interaction of the propulsion and steering complex with the ship's hull have not yet been sufficiently studied, even with regard to stable operation modes. Numerical values of coefficients of forces and moments are obtained mainly from the results of expensive experimental studies in experimental basins regarding propellers of a characteristic type and shape, under certain conditions of water flow around them. For other propellers, under other operating conditions, the values of these coefficients will differ. All this, in addition to the well-known problems of the interaction of the propulsive electric power plants, the propellers and the ship's hull, affects the accuracy of the calculation results.

The transition to a mathematical model in dimensionless units and the detection of generalized dimensionless parameters of the propulsive complex makes it possible to level out this shortcoming to some extent. All research is conducted precisely according to these – generalized parameters. If, in the process of research, the influence of changing the numerical values of these parameters on the behavior of the propulsive complex is analyzed, then in fact it is possible to obtain a set of ready-made solutions for any variant of their combinations. And if information about a more precise value of some coefficient appears in the future, it is enough to choose a ready-made solution from this totality.

Calculation results. The developed mathematical model of transient modes of operation allows analyzing the behavior of propulsive electric ships during maneuvers. An algorithm and a method for calculating the current values of mode indicators during the execution of various maneuvers by electric ships have been developed. The Runge-Kutta-Merson method is used to solve differential equations of transient processes. Based on the results of the calculation, the main quality indicators of all component parts of the complex are evaluated.

As an example, which confirms the efficiency of the developed model and calculation method, the simulation results of one of the maneuvers characteristic of electric ships are given below. The electric ship "Arcticaborg" was considered, the main technical data of which are presented in Table 2.

Table 2

Basic technical data of "Arcticaborg"

Length of the vessel according to k.v.l., m	65,1
Vessel speed, high school	13
Nominal parameters of heat engines	
Power, kW	2×1950
Rotation frequency, rpm	1000
Nominal parameters of propulsion electric motors	
Power, kW	2×1620 (Azipod)
Thruster	
Power, kW	150

The following maneuver was considered: acceleration of the electric ship on a straight course (with synchronous control of the propulsion electric motors) to a low speed $v = 0{,}3v_{nom}$ (at such speeds, the work of the thrusters is effective) and the output of the electric ship to circulation due to the inclusion (at the time $T = 15$) of the bow thrusters. The maneuver ends when time $T = 40$ is reached.

As a result of the calculation, the current values of the regime indicators of the complex were obtained. In particular, it is:

– components of vessel speed v_X, v_Y and angular velocity of its rotation Ω_Z;

– torque of heat engines M_D, moment of resistance M_G and current I_G of generators;

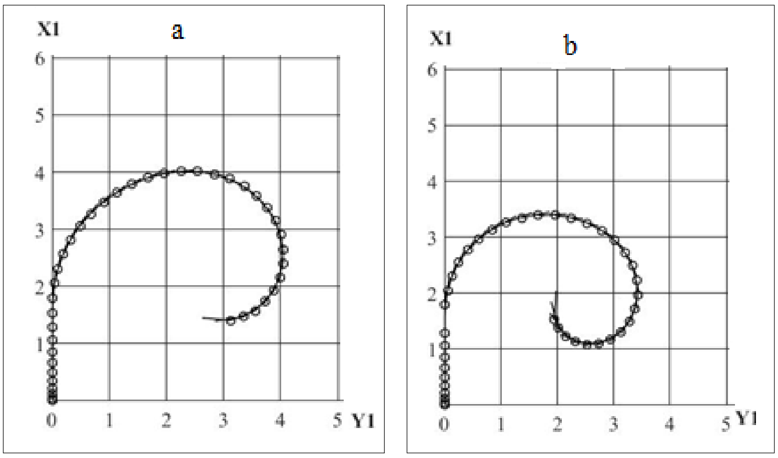
– current I_M, torque M_M and angular speed of rotation ω_M of propulsion electric motors;

– moment M_{Th} and angular speed of rotation ω_{Th} of *Th* electric motors.

According to the results of the calculations Fig. 6 shows the trajectories of the ship's movement on the coordinate plane of unrelated to the ship coordinate system $X_1 0 Y_1$ (the size of the grid is equal to the length of the ship L), at different values of the coefficient K_{TR}. The trajectories are built using the equations given in [12].

The Fig. 6 shows that with an increase in K_{TR} there is a decrease in the tactical diameter of circulation, extension and direct displacement of the vessel. In particular, the circulation diameter with $D_C = 4L$, with $K_{TR} = 0{,}15$, decreases to $D_C = 3{,}5L$, with

K_{TR} = 0,20. This indicates that the electric ship will be able to perform the maneuver in a smaller water area, that is, the maneuverability of the vessel improves.



a *b*

Figure 6 – Trajectory of the vessel movement at:
a – K_{TR} = 0,15; b – K_{TR} = 0,20

Fig. 7 shows the main operating parameters of the electrical power plant when performing the considered maneuver. The presented results correspond to the variant with K_{TR} = 0,15. For ease of understanding, they are grouped into separate drawings for: generator units (M_D, M_G, I_G); propulsion electric motors (I_M, M_M, ω_M); thruster electric motors (M_{Th}, ω_{Th}); vessel (v_X, v_Y, Ω_Z). All indicators are given in relative units (the line above the symbols – a sign of relative value – is omitted for the visual relief of the figure).

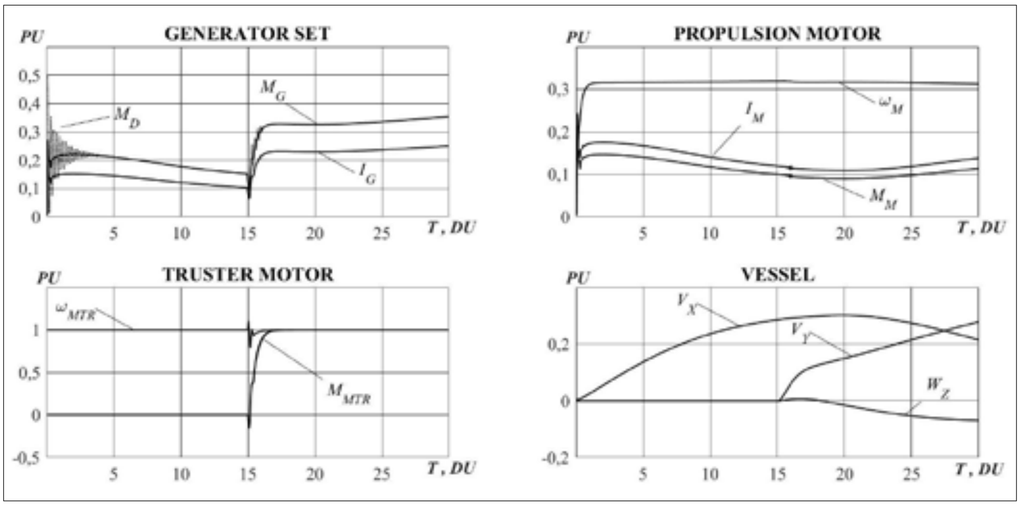


Fig. 7. Current values of mode indicators of the propulsive complex
when entering circulation for the K_{TR} = 0,15 option.

It can be seen from Fig. 7 that all the main mode indicators of generator units, electric propulsion motors, and thruster electric motors are within the permissible limits, which ensures the guarantee and safety of the maneuver.

Conclusions

A generalized mathematical model of transient modes of electric ships' propulsion complexes with thrusters on maneuvers has been developed. The model allows calculating the current values of the main mode indicators of the component parts of the complex during maneuvers and to evaluate the main indicators of the quality of maneuvering.

The system of dimensionless units, which is the basis of the construction of the model, allows leveling out the uncertainty in the assessment of external factors and the inaccuracy in determining the structural parameters of the complex. The obtained results will be acceptable for a wide class of vessels.

The performance of the developed mathematical apparatus is illustrated by the results of computer simulation of maneuvering modes. The model and the developed calculation method can be used both in the design of modern vessels with electric propulsion, and in the process of their operation to find the best ways to control electric propulsion.

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