UDC 519.865:338.518 DOI https://doi.org/10.33082/td.2022.3-14.05

TWO-PHASE MULTI-NOMENCLATURE PRODUCTION/STORAGE MODEL WITH RANDOM INPUT

M.Ya. Postan[1], S.N. Dashkovskiy[2], I.M. Moskvichenko[3]
[1]DSc, Head of the Department Management and Marketing,
Odessa National Maritime University, Odessa, Ukraine,
ORCID ID: 0000-0003-4891-3063
[2]DSc, Professor, Institute of Mathematics,
University of Wurzburg, Wurzburg, Germany,
ORCID ID: 0000-0001-7049-012X
[3]PhD, Associate Professor at the Department of Management and Marketing,
Odessa National Maritime University, Odessa, Ukraine

Summary

Introduction. *Recently, much attention in applications of stochastic models to real production/transportation systems modeling and control have been paid. It is explained by necessity to take into account the uncertainty and risks when projecting/operating such kind of systems.* ***Purpose****. To formulate and solve the problem of optimal values of products removal rates finding.* ***Results****. The mathematical model of a manufacturing system with several kinds of final product and one kind of raw materials is under consideration. This production system is interpreted as two-phase storage system with random input flow of raw materials which is described by Levy process with nondecreasing sample paths and zero drift. The first phase consists of one warehouse for raw materials storage and second one consists of industrial equipment and several parallel warehouses for final products storage. The production rates and rates of products removal from warehouses are constant.* ***Conclusions****. The joint distribution of storage levels of raw materials and any kind of final product at warehouses is found in closed form (in the terms of Laplace-Stieltjes transform). The problem of optimal values of products removal rates finding is formulated and solved.*

Key words: *Two-phase manufacturing system, random input, Levy process, probabilistic distribution.*

МОДЕЛЬ ДВОХФАЗОВОЇ БАГАТОНОМЕНКЛАТУРНОЇ ВИРОБНИЧО-СКЛАДСЬКОЇ СИСТЕМИ ІЗ ВИПАДКОВИМ ВХОДОМ

М.Я. Постан[1], С.М. Дашковський[2], І.М. Москвіченко[3]
[1]д.е.н., завідувач кафедри “Менеджмент і маркетинг”,
Одеський національний морський університет, Одеса, Україна,
ORCID ID: 0000-0003-4891-3063
[2]д.ф.-м.н., професор, Інститут математики,
Вюрцбурзький університет, Вюрцбург, Німеччина,
ORCID ID: 0000-0001-7049-012X
[3]к.е.н., доцент кафедри “Менеджмент і маркетинг”,
Одеський національний морський університет, Одеса, Україна

Анотація

Вступ. *Останні роки велика увага приділяється застосуванню стохастичних моделей для моделювання виробничо-транспортних систем та оптимального управління ними. Це викликано необхідністю враховувати фактори невизначеності та ризику в процесі проектування таких систем та управління ними.* ***Мета***. *Постановка та рішення проблеми визначення оптимальних значень інтенсивностей вивезення зі складу готової продукції.* ***Результати***. *Розроблена математична модель виробничої системи з декількома видами готової продукції та одним видом сировини. Виробнича система інтерпретується як двохфазова система збереження запасів з випадковим вхідним потоком сировини одного виду, який описується процесом Леві з неспадними траєкторіями та нульовим знесенням. Перша фаза включає склад для збереження сировини, а друга фаза включає виробниче обладнання та декілька паралельних складів для збереження готової продукції. Інтенсивності виробництва готової продукції, а також вивезення готової продукції зі складів, вважаються заданими та постійними.* ***Висновки***. *Знайдено в аналітичному вигляді (у термінах перетворення Лапласу-Стильтьєса) сумісний розподіл рівнів запасів сировини та готової продукції. Сформульована та вирішена проблема знаходження оптимальних значень інтенсивностей вивезення готової продукції зі складів.*

Ключові слова: *двофазна виробнича система, випадковий вхід, процес Леві, імовірнісний розподіл.*

1. Introduction

It is well-known that application of the Markov processes for modeling of different types of production/storage systems is motivated by necessity to take into account the following main factors of internal and external uncertainty:

a) random moments of the orders' giving from market to an enterprise for a product manufacturing;

b) irregularity of raw materials delivery to enterprise by transport;

c) restricted reliability of industrial equipment.

During the last 2-3 decades the much researches devoted to development and analysis of Markovian models of production/storage systems were fulfilled [1-6]. But it is almost impossible very often to study the real large-scale systems or networks by the analytical means because of dimension barrier. Therefore, it is expediently to develop the theoretical approaches which allow us to find out the conditions of full or partial decomposition of corresponding mathematical models. Typical example of such kind is the queueing networks theory [1; 2].

In our works [7–10] an approach was proposed for modeling the large-scale stochastic systems which is based on development of continuous variant of queueing networks theory – so-called stochastic storage networks theory. Generally speaking, this approach is a generalization of some classical stochastic storage models for the case of set of interacting single storehouses.

In this paper, the application of this analytic approach is demonstrated for modeling the two-phase production/storage system with several kinds of final product and random input of raw materials.

2. Description of mathematical model

Consider the following two-phase manufacturing system. The first phase consists of one warehouse for raw material storage. From this warehouse, if it isn't empty, the raw materials come to *M* machines (industrial equipment), located in parallel, for manufacturing corresponding final products. At the *m*-th machine the raw materials come with the rate W_m and with the same rate the *m*-th machine manufactures the final product of *m*-th kind. Correspondingly, the *m*-th final product comes to the warehouse (of second phase) with the rate W_m and with the rate $U_m < W_m$ it removes off (i.e. comes to consumers) if this warehouse isn't empty. The value U_m may be interpreted as the rate of *m*-th final product delivery to consumers. Once the storage level of raw materials at warehouse has been exhausted, then all machines have been stopped. It is assumed also that all warehouses have the unlimited capacities. This assumption, of course, is only simplification of reality but, from the mathematical point of view, analysis of a storage model with finite warehouses' capacities may be provided on the basis of this more simple case [10].

The input flow of raw materials is Levy process $X(t)$ [3] with nonnegative trajectories and zero drift, $X(0) = 0$. In our case $X(t)$ has the meaning of total input of raw materials during the time interval (0, *t*]. As it is known [3]

$$\mathsf{E}e^{-sX(t)} = e^{-\phi(s)t}, \mathrm{Re}\ s \geq 0, \tag{1}$$

where $\phi(s)$ is the cumulant of $X(t)$. For example, for compound Poisson process

$$\phi(s) = \lambda(1-\beta(s)),$$

where λ is the rate of the Poisson stream; $\beta(s) = \int_0^\infty e^{-st} dB(x), B(x)$ is the distribution function (d.f.) of jump sizes which is concentrated on $R_+ = [0,\infty)$. In this case process $X(t)$ may be interpreted in the following "logistical" way: through the exponentially distributed with mean $1/\lambda$ time intervals an enterprise receives an order from market for manufacturing a random quantities of final products. These quantities demand a batch of raw materials of random size with d.f. $B(x)$.

We denote by $\xi_1(t)$ the content of warehouse for raw materials storage at moment *t* and by $\xi_{2m}(t)$ the amount of the *m*-th final product at warehouse at moment *t*. Let $\xi_1(0), \xi_{1m}(0), m = 1,2,...,M$, are the non negative values.

According to the above assumptions the processes $\xi_1(t), \xi_{11}(t), ..., \xi_{1M}(t)$ satisfy the following system of integral equations (with probability 1):

$$\xi_1(t) = \xi_1(0) + X(t) - W(t - I_1(t)), \tag{2}$$

$$\xi_{2m}(t) = \xi_{2m}(0) + W_m(t - I_1(t)) - U_m(t - I_{2m}(t)), \quad m = \overline{1,M}, \tag{3}$$

where

$$I_1(t) = \int_0^t u(\xi(\tau))d\tau, I_{2m}(t) = \int_0^t u(\xi_1(\tau))u(\xi_{2m}(\tau))d\tau, m = \overline{1, M};$$

$$u(z) = 0, \text{if } z > 0, u(0) = 1; W = \sum_{m=1}^{M} W_m.$$

Note that $I_1(t), I_{2m}(t), m = \overline{1, M},$ are duration of time when the warehouses have remained the empty in the time interval (0, t].

The Eqs. (2), (3) are usual, for storage theory, balance relations describing input and output processes of material flows [3]. The conditions

$$W_m > U_m, m = \overline{1, M}, \tag{4}$$

are necessary to avoid the trivial situations, So, for example, if for any warehouse of second phase inequality (4) isn't valid, then this warehouse will become empty with probability 1 in a finite time and will remain empty after that.

3. Analysis of mathematical model

The main results concerning analysis of the model (2), (3) we formulate as the following theorems.

Theorem 1. *The system of integral equations* (2), (3) *has the unique non negative solution*

$$\xi_1(t) = \xi_1(0) + X(t) - Wt + WI_1(t), \tag{5}$$

$$\xi_{2m}(t) = \xi_{2m}(0) + V_m t - W_m I_1(t) + U_m I_{2m}(t), \tag{6}$$

where

$$WI_1(t) = \max\{0, -\xi_1(0) - \inf_{0 \le \tau \le t}[X(t) - Wt]\},$$

$$U_m I_{2m}(t) = \max\{0, -\xi_{2m}(0) - \alpha_m \xi_1(0) - \\ -\inf_{0 \le \tau \le t}[\alpha_m X(\tau) - U_m t]\}, m = \overline{1, M}; V_m = W_m - U_m; \\ \alpha_m = W_m / W.$$

The proof of Theorem 1 is based on reduction of Eqs. (2), (3) to classical storage model [3]. To prove it we multiply both sides of Eq. (2) by α_m, take the sum of Eqs.(2) and (3), and take into account the identity

$$u(\xi_1(t))u(\xi_{2m}(t)) = u(\alpha_m \xi_1(t) + \xi_{2m}(t)).$$

After these manipulations we arrive at the following equation for the process $\varsigma_m(t) = \xi_1(t) + \xi_{2m}(t)$:

$$\varsigma_m(t) = \varsigma_m(0) + \alpha_m X(t) - U_m(t - I_{2m}(t)). \tag{7}$$

The Eqs. (2) and (7) are the classical storage models with the Levy input processes $X(t)$, $\alpha_m X(t)$ and the release rates W, U_{2m} correspondingly. Applying the known results for classical storage model [3] we get the formulas (5) and (6).

The further results are related to determination of joint distribution of stochastic processes $\xi_1(t), \xi_{2m}(t), I_1(t), I_{2m}(t)$. Let us denote

$$\Psi_m^*(\mathbf{s}, É, \mathbf{x}; \omega) = \int_0^\infty \Psi_m(\mathbf{s}, É, \mathbf{x}; \omega) e^{-\omega t} dt, \operatorname{Re}\omega > 0,$$

where $\Psi_m(\mathbf{s}, É, \mathbf{x}; t) = \mathsf{E}\{\exp[-(\mathbf{s}, \xi(t)) - (É, \mathbf{I}_m(t)) | \xi(0) = \mathbf{x}\}$,

$\mathbf{s} = (s_1, s_2), É = (\omega_1, \omega_2), \mathbf{x} = (x_1, x_2) \geq \mathbf{0}$.

Theorem 2. *The Laplace transform of conditional joint distribution of random vectors* $\xi(t)$ *and* $\mathbf{I}_m(t)$ *is given by*

$$\Psi_m^*(\mathbf{s}, É, \mathbf{x}; \omega) = [e^{-(\mathbf{s},\mathbf{x})} - (s_1 W - s_2 W_m + \omega_1) H_{2m}^*(s_2, É, \mathbf{x}; \omega) -$$
$$-(s_2 U_m + \omega_2) H_{1m}^*(É, \mathbf{x}, \omega)][\omega + s_2 V_m - s_1 W + \phi(s_1)]^{-1}, \qquad (8)$$
$$\operatorname{Re}\omega > 0,\ \operatorname{Re} s_i > 0, \operatorname{Re}\omega_i > 0, i = 1, 2,$$

$$H_{1m}^*(É, \mathbf{x}; \omega) = \frac{\exp - (x_1 + x_2 / \alpha_m)\eta_{1m} - \omega_1 x_2 / W_m}{\omega_2 + U_m(\omega_1 + \eta_{1m} W) / W_m},$$

$$H_{2m}^*(s_2; É, \mathbf{x}; \omega) =$$
$$= [\exp(-s_2 x_2 - \eta_2 x_1) - (s_2 U_m + \omega_2) H_{1m}^*(É, \mathbf{x}; \omega)] \times$$
$$\times (W \eta_{2m} - W_m s_2 + \omega_1)^{-1},$$

$\eta_{1m} \equiv \eta_{1m}(\omega_1, \omega), \eta_{2m} \equiv \eta_{2m}(s_2, \omega)$ *are the unique continuous solutions to the functional equations*

$$\omega + \omega_1 V_m / W_m - U_m \eta_{1m} / \alpha_m + \phi(\eta_{1m}) = 0, \qquad (9)$$

$$\omega + s_2 V_m - W \eta_{2m} + \phi(\eta_{2m}) = 0 \qquad (10)$$

with the following properties:

$$a) \quad \frac{d}{d\omega} \eta_{1m}(0, \omega) \bigg|_{\omega = 0+} = \begin{cases} \alpha_m (U_m - \alpha_m \gamma)^{-1}, \alpha_m \gamma < U_m \\ \infty, \alpha_m \gamma = U_m, \end{cases}$$

$$\frac{\partial}{\partial \omega} \eta_{2m}(0, \omega) \bigg|_{\omega = 0+} = \begin{cases} (W - \gamma)^{-1}, \gamma < W \\ \infty, \quad \gamma = W, \end{cases}$$

where $\gamma = \phi'(0) = \mathrm{E}X(1) < \infty$; $\phi(s)$ *is the cumulant of X(t)* (*see* (1));

b) as $\omega \to 0+$, $\cdot_{1m}(0,\omega) \to \eta^0_{1m}$, *where* η^0_{1m} *is the la rgest positive root of the equation* $U_m \eta^0_{1m} = \alpha_m \phi(\eta^0_{1m})$ *and* $\eta^0_{1m} \geq 0$ *,iff* $\alpha_m \gamma > U_m$.

The proof of Theorem 2 may be given by analogy with the corresponding theorem in theory of classical storage model (2) or (7) [3]. Particularly, the formula (8) is derived by direct calculation of expression $\Psi^*_m(\mathbf{s}, \acute{E}, \mathbf{x}; \omega)$ taking into account the relations (5), (6).

Using the results of Theorems 1 and 2 we can immediately determine the conditions under which there exists a limit distribution of the random vector $(\xi_1(t), \xi_{2m}(t))$, as well as examine its asymptotic distribution. Setting in (8) $s_1 = s_2 = 0, \omega_1 = 0$ we obtain

$$\int_0^\infty e^{-\omega t} \mathsf{E}\{\exp(-\omega_2 I_{2m}(t)) \big| \xi_1(0) = x_1, \xi_2(0) = x_2\} dt = \\ = \frac{1}{\omega}[1 - \omega_2 e^{-(x_1 + x_2/\alpha_m)\eta_{1m}(0,\omega)} / (\omega_2 + \frac{U_m}{\alpha_m}\eta_{1m}(0,\omega))], \tag{11}$$

where $\eta_{1m}(0,\omega)$ is the root of functional Eq.(9) with $\omega_1 = 0$. By definition, $I_{2m}(t)$ is a monotone non decreasing function of *t*, therefore $I_{2m}(t) \to I_{2m} \leq \infty$ with probability one. Applying the Tauberian theorem, from (11) we get

$$\mathsf{E}\{\exp(-\omega_2 I_{2m})\} = 1 - \omega_2 \lim_{\omega \to 0+} \frac{e^{-(x_1 + x_2/\alpha_m)\eta_{1m}(0,\omega)}}{\omega_2 + U_m \eta_{1m}(0,\omega)/\alpha_m}.$$

The last expression equals to 0 if $\gamma \leq U_m / \alpha_m$, and equals to

$$1 - \omega_2 / (\omega_2 + U_m \eta^0_{1m} / \alpha_m) \tag{12}$$

if $\gamma > U_m / \alpha_m$.

Theorem 3. *a) Let* $\omega_1 = \omega_2 = 0, x_1 \geq 0, x_2 \geq 0$. *Then as* $t \to \infty$ *if* $\gamma > W$, *then* $\xi_1(t) \to \infty, \xi_{2m}(t) \to \infty$ *in distribution. If* $\alpha_m \gamma < U_m$, *then process* $(\xi_1(t), \xi_{2m}(t))$ *converges weakly to random vector* (ξ_1, ξ_{2m}) *with distribution given by the Laplace-Stieltjes transform*

$$\mathsf{E}\{e^{-s_1\xi_1-s_2\xi_{2m}}\}=(U_m-\gamma\alpha_m)\frac{s_2W}{s_1W-s_2V_m-\phi(s_1)}\times$$
$$\times\frac{s_1-\eta_{2m}(s_2,\ 0+)}{W_m s_2-W\eta_{2m}(s_2,\ 0+)},\qquad \mathrm{Re}\ s_i>0, i=1,2, \tag{13}$$

where $\eta_{2m}(s_2,0+)=\lim_{\omega\to 0+}\eta_{2m}(s_2,\omega)$.

b) If $\frac{U_m}{\alpha_m}<\gamma<W,\sigma^2<\infty,$

then $\lim_{t\to\infty}\mathsf{P}\{\frac{\xi_{2m}(t)-(\alpha_m\gamma-U_{2m})t}{\alpha_m\sigma\sqrt{t}}\le x\}=N(x),$

where *N(x) is standard normal distribution with zero mean and unit variance;* $\sigma^2=-\phi''(0)=\mathrm{Var}X(1).$

c) If $\gamma<U_m/\alpha_m,\sigma^2<\infty,$

then $\lim_{t\to\infty}\mathsf{P}\{\frac{U_m I_{2m}(t)-(U_m-\alpha_m\gamma)t}{\alpha_m\sigma\sqrt{t}}\le x\}=N(x).$

The proof of point a) of Theorem 3 is based on the formula (8) and application of the Tauberian theorem. To prove the point b) note that from (7), it follows

$$\frac{\xi_{2m}(t)-(\gamma\alpha_m-U_m)t}{\alpha_m\sigma\sqrt{t}}=\frac{\alpha_m\xi_1(0)+\xi_{2m}(0)}{\alpha_m\sigma\sqrt{t}}+\frac{X(t)-\gamma\ t}{\sigma\sqrt{t}}-$$
$$-\frac{\xi_1(t)}{\sigma\sqrt{t}}+\frac{U_m I_{2m}(t)}{\alpha_m\sigma\sqrt{t}}.$$

It is obvious that the first addend in right-hand side of the last relation tends to 0 as $t\to\infty$. If $\alpha_m\gamma>U_m$, then from (12) follows the existence of finite limit of $I_{2m}(t)$, therefore $I_{2m}(t)/\sqrt{t}\to 0$ in distribution. Since $\gamma<W$, there exists the limit distribution of process $\xi_1(t)$, hence $\xi_1(t)/\sqrt{t}\to 0$. Therefore

$$\lim_{t\to\infty}\mathsf{P}\{\frac{\xi_{2m}(t)-(\alpha_m\gamma-U_m)t}{\alpha_m\sigma\sqrt{t}}\le x\}=$$
$$=\lim_{t\to\infty}\mathsf{P}\{\frac{X(t)-\gamma\ t}{\sigma\sqrt{t}}\}.$$

But for the Levy process $X(t)$ the last limit equals to $N(x)$ [3]. The point c) of Theorem 3 may be proved similarly.

Note that $\varsigma_m(t)=0$ if $\xi_1(t)=\xi_{2m}(t)=0$. As it follows from theory of classical storage model and Eq. (7) if $\alpha_m\gamma<U_m$, then

$$\lim_{t\to\infty}\mathsf{P}\{\varsigma_m(t)=0\}=\lim_{t\to\infty}\mathsf{P}\{\xi_1(t)=0,\xi_{2m}(t)=0\}= \\ =1-\alpha_m\gamma/U_m.$$

Setting in (13) $s_1=0, s_2=s$ we have

$$f(s)\equiv \mathrm{E}\{e^{-s\xi_{2m}}\}=\frac{U_m}{V_m}(1-\frac{\alpha_m\gamma}{U_m})\frac{W\eta_{2m}(s,0+)}{W\eta_{2m}(s,0+)-sW.} \tag{14}$$

Using this formula we can calculate the stationary mean and variance:

$$\mathsf{E}\xi_{2m}=-f'(0)=\frac{V_m\alpha_m\sigma^2}{2(W-\gamma)(U_{2m}-\alpha_m\gamma)}, \tag{15}$$

$$\mathsf{Var}\xi_{2m}=f''(0)-(f'(0))^2=\frac{\alpha_m V_m^3}{6(W-\gamma)(U_m-\alpha_m\gamma)^2}\times \\ \times\{2\sigma^4(W_m+U_m-2\alpha_m\gamma)+2\phi'''(0)(W-\gamma)(U_m-\alpha_m\gamma)\}- \\ -(E\xi_{2m})^2.$$

For example, if $\phi(s)=\lambda(1-\beta(s))$, then $\phi^{(k)}(0)=(-1)^{k+1}\lambda\beta^{(k)}$, where $\beta^{(k)}, k=1,2,3,$ are the first three initial moments of d.f. $B(x)$.

These results may be used for approximate determination of warehouses' capacities and formulation of some optimization problems.

For example, let us it is required to find out the values $U_m, m=\overline{1,M}$, that minimize mean current cost for storage of final products and costs for their transportation $(\bar{E})$. The objective function is given by

$$\bar{E}=\sum_{m=1}^{M}(e_m U_m+c_m\mathsf{E}\xi_{2m}), \tag{16}$$

where e_m is transportation cost for delivery of 1 t of m-th final product to consumers; c_m is storage expenses per unit of time for 1 t of m-th final product. Taking into account the formula (15) we can re- write the expression (16)

$$\bar{E}=\sum_{m=1}^{M}e_m U_m+\frac{\sigma^2}{2(W-\gamma)}\sum_{m=1}^{M}\frac{c_m\alpha_m(W_m-U_m)}{U_m-\alpha_m\gamma}\ . \tag{17}$$

We assume that the following condition hold true:

$$\sum_{m=1}^{M} \mathsf{E}\xi_{2m} = \frac{\sigma^2}{2(W-\gamma)} \sum_{m=1}^{M} \frac{\alpha_m (W_m - U_m)}{U_m - \alpha_m \gamma} \le C, \tag{18}$$

where C is the given total capacity of warehouses for final products storage. The additional constraints are just the stability conditions

$$U_m < \alpha_m \gamma, m = \overline{1, M}. \tag{19}$$

The stochastic optimization problem (17)-(19) may easily be solved by the Lagrange multipliers method. Its solution is given by

$$U_m = W_m [\ \frac{\gamma}{W} + \sigma \sqrt{\frac{c_m + \mu}{2e_m W_m W}}\ ], \qquad m = \overline{1, M},$$

where μ is Lagrange multiplier which may be found as the unique root of the equation

$$\sum_{m=1}^{M} \alpha_m \sqrt{e_m W_m / (c_m + \mu)} = \sigma \sqrt{W/2} [1 + 2C(W-\gamma)/\sigma^2]/(W-\gamma).$$

4. Conclusions

The results obtained show that it is possible to investigate by analytical means the stochastic models of storage networks on the basis of classical models for single storehouse. The further development of our approach may concern the following directions:

a) Consideration of several kinds of raw materials; but here we must take into account the following circumstance: once a warehouse for storage of any kind of raw materials has been empty, then corresponding technological lines (machines) which process these raw materials must be stopped.

b) Taking into account the irregularity of raw materials delivery at enterprise and random fluctuation of demand for final product. Here, it is possible to use the mathematical technique given in the book [3] (Markov-compound Poisson input and Markov-modulated demand).

c) Investigation of models of manufacturing systems by representation their as stochastic storage networks with more complex topology.

REFERENCES

1. Kelly, F.P. (1979). Reversibility and Stochastic Networks. New York, J. Wiley & Sons.
2. Neuts, M.F. (1995). Matrix-Geometric Solutions in Stochastic Models: An Algorithmic Approach. Boston, 2nd Ed. Dover Publ.
3. Kulkarni, V.G. (1995). Modeling and Analysis of Stochastic Systems. London, Chapmen & Hall.
4. Prabhu, N.U. (1998). Stochastic Storage Processes: Queues, Insurance Risk, Dams, and Data Communication. New York, 2nd Ed. Springer Verlag.

5. Ost, A. (2001). Performance of Communication Systems – A Model-Based Approach with Matrix-Geometric Methods. Berlin, Springer Verlag.
6. Ryzhikov, Yu.I. (2001). Queueing Theory and Inventory Control [Teoriya ocheredej i upravlenie zapasami]. St.-Petersburg [in Russian], 384 p.
7. Voevudskiy, E.N., & Postan, M.Ya. (1991). Stochastic Model of Multi-Phase Storage System. Communications of Academy of Science of USSR. 318(1), 5156.
8. Postan, M.Ya. (1995). On Application of Stochastic Processes of the Stock Theory to Modeling Computerized Information Networks. Prob. Inform. Transm., 31(3), 262–283.
9. Postan, M.Ya. (2000). Theory of Stochastic Storage Networks: Present State and Perspectives. In: Proc. of 16th IMACS World Congress 2000 on Scientific Computation, Applied Mathematics, and Simulation, CD, File CP 216-5. Lausanne.
10. Postan, M.Ya. (2006). Economic-Mathematical Models of Multimodal Transport [Ekonomiko-matematicheskie modeli smeshannyh perevozok]. Odessa, Astroprint [in Russian], 376 p.