

**MODEL FOR OPTIMAL PLANNING OF MARKETING ACTIVITY
OF PLANT, PRODUCTION AND TRANSPORTATION
OF PERISHABLE PRODUCTS**

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Summary

Introduction. During the last decade in theory of logistics and its applications the great attention was paid to the problem of perishable products flows control that was inspired by foods production/transportation growth. Due to importance of this problem even special direction was formed in logistics, so called “cold logistics” which studies warehousing/transportation problems of perishable items taking into account the special regimes of their storage. **The aim** of this paper is the further development of approach pointed out above and development of dynamic optimization model for the case of perishable material and finished perishable product coming through the logistic chain taking into account the control of deterioration process at warehouses and increasing the demand for finished product via additional costs for marketing activity. **Results.** In the article, the dynamic optimization model for planning of raw materials supply, production of perishable finished product and its transportation to destinations is proposed. It is assumed that the additional investments intended for decreasing of raw materials and finished perishable product’s deterioration during its warehousing are provided. The above model is based on the Wagner-Whitin type model in the inventory control theory and classical transportation problem. The objective is to maximize the total profit of supply chain over the given planning horizon. Two cases are considered in details: 1) demand at destinations is given and fixed; 2) demand is controlled by additional investments. **Conclusions.** Our approach allows to increasing the total profit of logistic system due to additional expenses directed on reduction of deterioration of perishable goods. The calculated results may be useful for logistics operators and other participants of logistic cold chain (e. g. warehouses operators, transport companies). The proposed optimization models are relatively simple and may be implemented in practice with the standard software. In order to conduct calculations it is the necessary to collect the relevant data and to create appropriate databases. The results obtained may be used as the basis for our further investigation and research in the field of “cold logistics”.

Key words: supply, production, transportation, perishable finished product, planning, dynamic optimization, marketing, control of deterioration.

МОДЕЛЬ ОПТИМАЛЬНОГО ПЛАНУВАННЯ
МАРКЕТИНГОВОЇ ДІЯЛЬНОСТІ ПІДПРИЄМСТВА, ВИРОБНИЦТВА
ТА ТРАНСПОРТУВАННЯ ШВИДКОПСУВНОЇ ПРОДУКЦІЇ

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Анотація

Вступ. Протягом останнього десятиліття в теорії логістики та її розробок велика увага приділялася проблемі контролю потоків швидкопсувних продуктів, яка була викликана зростанням виробництва/транспортування харчових продуктів. Враховуючи важливість цього питання, в логістиці навіть сформувався особливий напрямок, так звана «холодна логістика», яка вивчає проблеми складування/транспортування швидкопсувної продукції з урахуванням особливих режимів їх зберігання. **Мета** цієї статті – подальший розвиток зазначеного вище підходу та розробка моделі динамічної оптимізації швидкопсувного матеріалу та готового швидкопсувного продукту, що потрапляє до логістичного ланцюгу з урахуванням контролю процесу псування на складах та збільшення попиту на готовий продукт за рахунок додаткових маркетингових витрат. **Результати.** У статті запропоновано модель динамічної оптимізації планування постачання сировини, виробництва готової швидкопсувної продукції та її транспортування в пункти призначення. Вона також передбачає додаткові інвестиції, спрямовані на зменшення випадків псування сировини та готової швидкопсувної продукції під час її зберігання на складах. Вищезазначена модель заснована на моделі Вагнера-Уїтіна в теорії управління запасами та класичній транспортній задачі. Задача – максимізувати загальний прибуток ланцюга поставок за заданий горизонт планування. Автори детально розглядають два кейса: 1) попит задається і фіксується у пунктах прибуття; 2) попит контролюється додатковими інвестиціями. **Висновки.** Наш підхід дозволяє збільшити загальний прибуток логістичної системи за рахунок додаткових витрат, спрямованих на зменшення псування швидкопсувних товарів. Результати розрахунків можуть бути корисними для логістичних операторів та інших учасників логістичного холодового ланцюга (наприклад, операторів складів, транспортних компаній). Запропоновані оптимізаційні моделі відносно прості і можуть бути реалізовані на практиці за допомогою стандартного програмного забезпечення. Для проведення розрахунків необхідно зібрати відповідні дані та створити бази даних. Отримані результати можуть бути покладені в основу наших подальших розвідок та досліджень у сфері «холодної логістики».

Ключові слова: постачання, виробництво, транспортування, швидкопсувна готова продукція, планування, динамічна оптимізація, маркетинг, контроль псування.

Introduction. During the last decade in theory of logistics and its applications the great attention was paid to the problem of perishable products flows control that was inspired by foods production/transportation growth. Due to importance of this problem even special direction was formed in logistics, so called “cold logistics” which studies warehousing/transportation problems of perishable items taking into account the special regimes of their storage [1–3]. Due to the report presented by the Food and Agriculture Organization of the United Nations (FAO) [4] roughly one third of the food produced for human consumption in the world gets lost or wasted (approximately 1.3 billion tones every year). These processes are observed throughout all stages of the supply chain, from initial production up to final consumption. These losses are high both in industrialized and developing countries. More than 40 % of the food losses take place at post harvest and processing levels in developing countries. In industrialized countries, the majority of food losses occur at retail and consumer levels (more than 40 % of overall losses). It is well-known that inventory control theory plays an important role in logistical applications. Indeed, integrated logistical management first of all is intended for development of optimal supply plan, work-in-process and production plans, as well as optimal transportation plans for perishable finished product delivery to destination. At the same time the known models from inventory control theory cannot be applied in the “cold logistics” practices immediately. In many real situations arising in logistical management it is needed to adopt and generalize the classical models of inventory control theory for the case of deterioration of perishable materials and finished product under prolonged warehousing. Deterioration can deal with products spoilage, physical depletion, gradual loss of qualitative properties of materials with the passage of time and above all storage conditions changing [5; 6]. It is naturally to suppose that the volume of perishable products deterioration depends: a) on technical characteristics of refrigerating equipment and corresponding cost directed on supporting of special warehousing regimes; b) on additional costs directed on increasing the demand via additional costs for marketing activity of the plant.

In the articles [7–9] the simple models for optimal lot sizing of perishable product based on generalization of the classical Wilson model were studied. However, in the cited works the possibility of warehousing regime control was not considered. In the studies [10; 11] the models were proposed for optimal planning of integrated logistic chains functioning including supply of materials, manufacturing of perishable finished product and its delivery at points of destination based on generalization of the Wagner-Whitin model from inventory control theory.

The aim of this paper is the further development of approach pointed out above and development of dynamic optimization model for the case of perishable material and finished perishable product coming through the logistic chain taking into account the control of deterioration process at warehouses and increasing the demand for finished product via additional costs for marketing activity. This idea was firstly mentioned in our previous works [12–14].

Main results of investigation. Our first target is construction the corresponding mathematical model for solving the problem under consideration. Firstly, we consider more simple case of fixed demand.

Let us consider a plant which produces the K types of perishable finished product subjected to deterioration under storage at the plant’s warehouse. To manufacture these

products the R kinds of material and complete set are used which are subjected to deterioration during their storage, as well. It is assumed that the matrix

$$A = \|a_{rk}\|, k = 1, 2, \dots, K; r = 1, 2, \dots, R,$$

of technological coefficients is given, where a_{rk} is the amount of the r -th kind of material needed for manufacturing of the k -th type of perishable finished product's unit.

A plant purchases all kinds of materials at the R suppliers. The finished perishable products must be shipped to the N destinations. The planning horizon is T (time is measured in discrete units). The total demand for the k -th type of perishable finished product at the n -th destination over the period T is known and equals to $d_{kn} > 0$ (it may be determined, for example, in result of market's analysis). Taking into account the given demand, plant purchases the materials and manufactures the products (See Figure).

In addition, we make the following assumptions:

- The market of materials is unlimited.
- All ordering of materials and delivering of finished perishable products occurs at the start of each period. Inventories of materials are charged on the amount on hand in the end of each period.
- The lead time is zero; that is, an order arrives as soon as it is placed.
- The time of transportation of any amount of perishable finished product to any destination doesn't depend on this amount.
- The production equipment is absolutely reliable.
- The capacities of production lines of plant are limited only by capacities of warehouses' for storage of materials and finished perishable products.

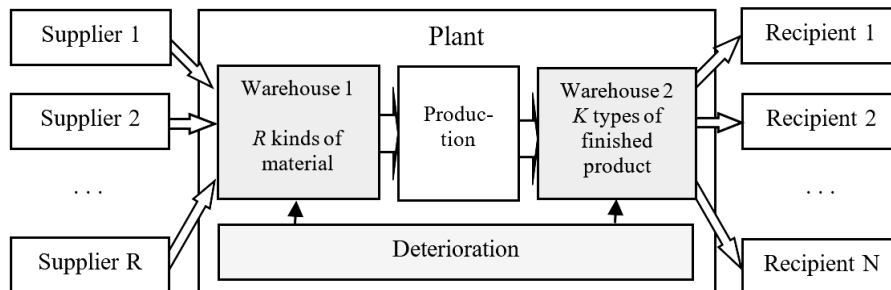


Figure. The part of logistic chain of perishable product (Source: own research)

Similarly as in the article [10] for better understanding, we introduce the following designations:

- Let x_{rt} be the amount of the r -th kind of material ordered and purchased in period t , for $t = 1, 2, \dots, T$.
- Let y_{kt} be the amount of the k -th type of perishable finished product which plant plans for output in the end of period t , for $t = 1, 2, \dots, T$.
- Let z_{knt} be the amount of the k -th type of perishable finished product planned for delivery to the n -th destination in the end of period t , for $t = 1, 2, \dots, T$.
- Let s_{knt} be the sale unit price for the k -th type of perishable finished product shipped to the n -th destination in period t , for $t = 1, 2, \dots, T$.
- Let P_{rt} be the per unit order cost and K_{rt} be the fixed order cost for the r -th kind of material ordered in period t , for $t = 1, 2, \dots, T$.

- Let e_{kt} be the per unit production cost of the k -th type of perishable finished product in period t , for $t = 1, 2, \dots, T$.
- Let c_{knt} be the cost of transportation of the unit of the k -th type of perishable finished product from plant to the n -th destination in period t , for $t = 1, 2, \dots, T$.
- Let $h_{1rt}(h_{2kt})$ be the holding cost per unit of the r -th kind of material (of the k -th type of perishable finished product) in period t , for $t = 1, 2, \dots, T$.
- Let $C_1(C_2)$ be the warehouse's capacity for storage of materials (perishable finished products).
- Let $q_{1r}(q_{2k})$ be the initial inventory level of the r -th kind of material (of the r -th type of perishable finished product). It is assumed that

$$\sum_{r=1}^R q_{1r} \leq C_1, \quad \sum_{k=1}^K q_{2k} \leq C_2.$$

- Let $I_{1rt}(I_{2kt})$ be the inventory level of the r -th kind of material (of the k -th type of perishable finished product) in the end of period t , for $t = 1, 2, \dots, T$.
- Let us $0 \leq \beta_{1r}, \beta_{2k} \leq 1$ are the coefficients describing the deterioration of the r -th kind of material and the k -th kind of perishable finished product during their storage at warehouses correspondingly.
- It is assumed that during the delivery of material to a plant and delivery of perishable finished product at destinations there are not subject to any deterioration. To avoid the trivial situations, we shall suppose that

$$\sum_{n=1}^N d_{kn} > q_{2k}, \quad k = 1, 2, \dots, K.$$

It is obvious that the following inventory-balanced equations are valid:

$$I_{1rt} = (1 - \beta_{1r})I_{1r,t-1} + x_{rt} - \sum_{k=1}^K a_{rk}y_{kt}, \quad r = 1, 2, \dots, R, \quad (1)$$

$$I_{2kt} = (1 - \beta_{2k})I_{2k,t-1} + y_{kt} - \sum_{n=1}^N z_{knj}, \quad k = 1, 2, \dots, K; \quad t = 1, 2, \dots, T, \quad (2)$$

where $I_{1r0} = q_{1r}, I_{2k0} = q_{2k}$.

From (1), (2), it follows the relations

$$I_{1rt} = (1 - \beta_{1r})^t [q_{1r} + \sum_{j=1}^t (x_{rj} - \sum_{k=1}^K a_{rk}y_{kj}) / (1 - \beta_{1r})^{t-j}], \quad (3)$$

$$r = 1, 2, \dots, R,$$

$$I_{2kt} = (1 - \beta_{2k})^t [q_{2k} + \sum_{j=1}^t (y_{kj} - \sum_{n=1}^N z_{knj}) / (1 - \beta_{2k})^{t-j}], \quad (4)$$

$$k = 1, 2, \dots, K.$$

Since

$$\sum_{r=1}^R I_{1rt} \leq C_1, \quad \sum_{k=1}^K I_{2kt} \leq C_2, \quad t = 1, 2, \dots, T,$$

then from (3), (4), we obtain

$$\sum_{r=1}^R (1 - \beta_{1r})^t q_{1r} + \sum_{r=1}^R \sum_{j=1}^t (1 - \beta_{1r})^{-j} x_{rj} - \sum_{j=1}^t \sum_{r=1}^R \sum_{k=1}^K (1 - \beta_{1r})^{-j} a_{rk}y_{kj} \leq C_1, \quad (5)$$

$$\begin{aligned} & \sum_{k=1}^K (1 - \beta_{2k})^t q_{2k} + \sum_{k=1}^K \sum_{j=1}^t (1 - \beta_{2k})^{-j} y_{kj} - \\ & - \sum_{k=1}^K \sum_{n=1}^N \sum_{j=1}^t (1 - \beta_{2k})^{-j} z_{knj} \leq C_2. \end{aligned} \quad (6)$$

On the other hand, in period t it cannot be consumed the r -th material and delivered the k -th product in amounts more than inventory levels $I_{1,r,t-1}$ and $I_{2,k,t-1}$ correspondingly in the end of period $t-1$, that is:

$$\begin{aligned} & \sum_{k=1}^K a_{rk} y_{kt} \leq I_{1,r,t-1}, \quad r = 1, 2, \dots, R, \\ & \sum_{n=1}^N z_{knt} \leq I_{2,k,t-1}, \quad k = 1, 2, \dots, K, \end{aligned}$$

Therefore, from (5), (6) we get

$$\sum_{k=1}^K a_{rk} y_{rt} + \sum_{j=1}^{t-1} \sum_{k=1}^K a_{rk} y_{kj} (1 - \beta_{1r})^j \leq q_{1r} (1 - \beta_{1r})^{t-1} + \sum_{j=1}^{t-1} x_{rj} (1 - \beta_{1r})^j, \quad r = 1, 2, \dots, R, \quad (7)$$

$$\begin{aligned} & \sum_{n=1}^N z_{knt} + \sum_{j=1}^{t-1} \sum_{n=1}^N z_{knj} (1 - \beta_{2k})^j \leq q_{2k} (1 - \beta_{2k})^{t-1} + \\ & + \sum_{j=1}^{t-1} y_{kj} (1 - \beta_{2k})^{t-1}, \quad k = 1, 2, \dots, K; \quad t = 1, 2, \dots, T. \end{aligned} \quad (8)$$

At last, the perishable finished product of the k -th kind must be delivered at the n -th destination in amount d_{kn} over the planning horizon, that is

$$\sum_{t=1}^T z_{knt} = d_{kn}, \quad n = 1, 2, \dots, N; \quad k = 1, 2, \dots, K. \quad (9)$$

In the model described above the coefficients of materials and finished perishable product deterioration β_{1r} , β_{2k} may be considered as control variables, as well. Indeed, generally speaking, they may depend on power of refrigerating equipment of plant or, in value expression, on this equipment price. Denote V_1 and V_2 the values of refrigerating equipment at warehouses for material and finished product correspondingly. It is naturally to suppose that $\beta_{1r}(V_1)$, $\beta_{2k}(V_2)$ are the non-increasing functions of their variables satisfying the following conditions

$$\beta_{1r}(\infty) = 0, \beta_{2k}(\infty) = 0, \quad \beta_{1r}(0) = 1, \beta_{2k}(0) = 1. \quad (10)$$

The simplest dependencies of such kind, for example, are

$$\beta_{1r}(V_1) = \frac{\beta_{1r}}{(1 + \mu_{1r} V_1)^{\alpha_r}}, \quad \beta_{2k}(V_2) = \frac{\beta_{2k}}{(1 + \mu_{2k} V_2)^{\gamma_k}}, \quad (11)$$

where $\mu_{1r}, \mu_{2k}, \alpha_r, \gamma_k$ are the positive coefficients determining by methods of mathematical statistics; β_{1r}, β_{2k} are the deteriorating coefficients reflecting basic deterioration without additional cost for its reduction.

Note that according to above designations the expressions $p_{rt} \beta_{1r} I_{1rt}$ and $s_{knt} \beta_{2k} I_{2kt}$ have the meaning of economic losses caused by deterioration of the r -th kind of materials and the k -th kind of finished perishable product during their storage in period t correspondingly.

The demand d_{kn} in the right-hand side of relation (9) may also be considered as control value. Let us suppose that d_{kn} is the a no decreasing function of values V_3 . The possible dependence d_{kn} on V_3 may be, for example, as follows

$$d_{kn}(V_3) = d_{kn}(1 + \lambda_{kn} V_3)^s, s \geq 0,$$

where d_{kn} is demand for the case $V_3 = 0$ (i. e. without additional cost for marketing).

Expression for total profit of all integrated logistical chain (i. e. objective function) taking into account the cost for control of refrigerating regimes is

$$\begin{aligned}
 P = & \sum_{t=1}^T \left\{ \sum_{n=1}^N \sum_{k=1}^K p_{knt} z_{knt} - \sum_{k=1}^K [e_{kt} y_{kt} + (h_{2kt} + s_{knt} \beta_{2k}) ((1 - \beta_{2k})^t q_{2k} + \right. \\
 & + \sum_{j=1}^t (1 - \beta_{2k})^{-j} y_{kt} - \sum_{n=1}^N \sum_{j=1}^t (1 - \beta_{2k})^{-j} z_{knj})] - \\
 & - \sum_{r=1}^R [p_{rt} x_{rt} + K_{rt} \delta(x_{rt}) + (h_{1rt} + p_{rt} \beta_{1r}) ((1 - \beta_{1r})^{-j} q_{1r} + \\
 & \left. + \sum_{j=1}^t (1 - \beta_{1r})^{-j} x_{rt} - \sum_{k=1}^K \sum_{j=1}^t (1 - \beta_{2r})^{-j} a_{rk} y_{kt})] \right\} - V_1 - V_2 - V_3,
 \end{aligned} \tag{12}$$

where $p_{knt} = s_{knt} t - c_{knt}$; $\delta(x) = 1$, if $x > 0$, $\delta(0) = 0$.

The optimization problem may be now formulated by the following way: it is needed to find out the nonnegative values of variables $x_{rt}, y_{kt}, z_{knt}, V_1, V_2, V_3$ satisfying the conditions (5)–(9), (11) and maximizing the function (12). This optimization problem may be solved by the method of dynamic programming [14]. The other method of its solving is based on the method proposed in the work [15]. This method allows to eliminate from consideration the non-differentiable term $K_{rt} \delta(x_{rt})$ entering the function (12) by introduction of additional variables.

Conclusions

In this paper, we proposed the approach to modeling and optimization of integrated logistics system functioning for the case of perishable materials and finished perishable goods which is based on inventory control theory application. The main idea of our approach is coordination among supply firm, plant, and transport companies at the stage of their joint plans development over the finite planning horizon. Our approach allows to increasing the total profit of logistic system due to additional expenses directed on reduction of deterioration of perishable goods. The calculated results may be useful for logistics operators and other participants of logistic cold chain (e. g. warehouses operators, transport companies). The proposed optimization models are relatively simple and may be implemented in practice with the standard software. In order to conduct calculations it is the necessary to collect the relevant data and to create appropriate databases. The results obtained may be used as the basis for our further investigation and research in the field of “cold logistics”.

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