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MODEL APPROACH IN PROJECT MANAGEMENT METHODOLOGY

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Summary

Introduction. *Some mathematic models of project management encounter the need for determining the maximum radius of hypersphere immersed into a polyhedral region. The modern mathematic tools of the optimization theory in conjunction with application of computer technology allows for solving nonlinear optimization problems. However, linearization of complicated nonlinear problems keeps being permanently feasible. Such a simplification enables using exact classic methods of optimization solution as opposed to approximate ones for nonlinear optimization. We have to set a task of rigorous mathematic reduction (linearization) of a polydimensional nonlinear optimization problem on immersion of a maximum-radius hypersphere into a convex polyhedral region. Let us have a closed polyhedron provided by a system of linear algebraic inequalities. The maximum-radius hypersphere is to be placed into the closed polyhedron region.* ***Purpose.*** *The article provides analysis of a model on determining the maximum radius of hypersphere placed (immersed) into a polyhedral region (a convex set restricted by straight lines) that enables taking account of a big set of factors including the following: Project Integration Management; Project Scope Management, Project Quality Management Project Time Management, Project Cost Management, Project Communication Management, Project Procurement Management and Project Risk Management.* ***Result.*** *The model has proposed rigorous mathematic reduction (linearization) of a nonlinear optimization problem on placing a maximum-radius hypersphere into a convex polyhedral region, to a linear optimization problem. Thus, the problem on placing the maximum-radius hypersphere within a polyhedron shall be formulated as a linear optimization problem.* ***Conclusions.*** *It has been rigorously proven that the problem on immersing the maximum-radius hypersphere into a polyhedron can be represented as a linearization problem. The problem has been reduced to a classical linear optimization problem soluble by known methods. The proposed approach is generalized on an arbitrary finite dimensionality problem.*

Key words: *project lifecycle, hypersphere, polyhedron, linear optimization.*

МОДЕЛЬНИЙ ПІДХІД У МЕТОДОЛОГІЇ УПРАВЛІННЯ ПРОЕКТАМИ

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Анотація

Вступ. *У деяких математичних моделях управління проектами виникає потреба встановлення максимального радіусу гіперсфери, зануреної в поліедральну галузь. Сучасний математичний апарат теорії оптимізації сумісно із застосуванням комп'ютерних технологій дає змогу розв'язувати нелінійні задачі оптимізації, але завжди існує доцільність лінеаризації складних нелінійних задач. Таке спрощення дає змогу використовувати точні класичні методи оптимізаційного розв'язку, на відміну від наближених, для нелінійної оптимізації. Поставимо завдання строгого математичного зведення (лінеаризації) багатовимірної нелінійної задачі оптимізації про занурення гіперсфери максимального радіусу в опуклу ділянку типу поліедру. Нехай маємо замкнений поліедр, поданий системою лінійних алгебраїчних нерівностей. У ділянці замкненого поліедру необхідно розмістити гіперсферу максимального радіусу.* ***Мета.*** *У статті проаналізовано модель установлення максимального радіусу гіперсфери, розміщеної (зануреної) у поліедральну ділянку (опуклу множину, обмежену прямими лініями), яка забезпечує врахування великої множини факторів, серед яких – управління інтеграцією (Project Integration Management); предметна ділянка проекту (Project Scope Management); управління якістю (Project Quality Management); управління часом (Project Time Management); управління вартістю (Project Cost Management); управління комунікаціями (Project Communication Management); управління контрактами (Project Procurement Management); управління ризиками (Project Risk Management).* ***Результати.*** *У моделі запропоновано строге математичне зведення (лінеаризація) нелінійної оптимізаційної задачі про розміщення гіперсфери максимального радіусу в опуклу ділянку типу поліедру до задачі лінійної оптимізації. Таким чином, задача про розміщення гіперсфери найбільшого радіусу в поліедрі формулюється як задача лінійної оптимізації.* ***Висновки.*** *Строго доведено можливість лінеаризації задачі про занурення гіперсфери максимального радіусу у поліедр. Задачу зведено до класичної задачі лінійної оптимізації, яка може бути розв'язана відомими методами. Запропонований підхід узагальнюється на задачі довільної скінченої вимірності.*

Ключові слова: *життєвий цикл проекту, гіперсфера, поліедр, лінійна оптимізація.*

Introduction. In the modern information society, most ideas are implemented through the use of the project management methodology. This specific approach ensures reasonability of both the time and the resources expenditure. The project management methodology keeps being developed and improved. The range of project management methods is rather wide. The model approach is ever more frequently included in the methodology.

Problem statement. Many mathematic models describing stages of project management encounter the task of mandatory use of as many certain factors as possible.

Modeling of lifecycle of a project, particularly of a project on using agile methodologies, frequently encounters the need for taking account of a set of factors a part of which is determined in the PMI PMBOK standard as 9 function areas of the management:

1) Project Integration Management;
2) Project Scope Management;
3) Project Quality Management;
4) Project Time Management;
5) Project Cost Management;
7) Project Communication Management;
8) Project Procurement Management (contracts) and;
9) Project Risk Management.

This approach contributes to avoiding a sudden and premature finish of a project without achieving its objective, except for cases when it is decided to stop implementing a project prior to its scheduled finish.

In this situation, we can see appearance of the need to resolve the problem of maximizing the coverage of the project management area with such factors, provided that there are some effective constraints. Such approach can be modeled as determining the maximum radius of a hypersphere placed (immersed) into the polyhedral region (a convex set restricted by straight lines).

The state of the art review. Modeling in the area of project management is used in many researches. Thus, the article proposes a model of evaluating competences of organizations acting in project, program and project portfolio management [1].

The model of efficient formalization of processes interacts on the project level. The effect of project portfolio complexity on the procedure of such formalization has been analyzed in the article on the level of portfolio [2].

At the same time, modern mathematical tools of the optimization theory [3; 4; 9] in conjunction with using computer technology allows for obtaining approximate solution of nonlinear problems. However, it is always reasonable to linearize complex nonlinear problems. Such simplification allows for using accurate classic methods of optimization solution as opposed to approximated ones for nonlinear optimization [10; 16].

Research objective. The objective of the research consisted in rigorous mathematic reduction (linearization) of a nonlinear optimization problem on placing a maximum-radius hypersphere into a convex polyhedral region, to a linear optimization problem.

Statement of basic material. Let us consider the general statement of the problem. Development of an IT project provides for formulation of hypotheses each of which is to be described and analyzed, contributing to adaptation of the project to customer's requirements and taking account of as many factors as possible to facilitate its efficient implementation. The stage of project formation includes grounding the need for involving a set of factors containing, in addition to those indicated above (according to PMI PMBOK standard), the following: possibility of technical realization, environmental impact, market efficiency, institutional acceptability, social aspects, financial and economic value. It is a difficult problem to determine the elements of such a set. For its solution, it has been proposed to use a problem on placing a maximum-radius hypersphere into convex polyhedral region, reduced to a linear optimization problem.

In the finitely measurable real linear space $\mathbb{R}^n$ consisting of a set of factors having effect on efficient implementation of the project, let us have a closed polyhedron Ω_I provided by a system of linear algebraic inequalities

$$\Omega_\mathrm{I}: \begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \le b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \le b_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots, \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \le b_m, \end{cases}$$
$$x_1 \ge 0,\ x_2 \ge 0,\ \ldots,\ x_n \ge 0.$$

In the region $\Omega_\mathrm{I} \subset \mathbb{R}^n$, we need to place the hypersphere with the biggest radius. For $\mathbb{R}^2$, it will be a circle, and for the three-dimensional space $\mathbb{R}^3$ – it will be a sphere. For a two-dimensional case, let us perform a graphic interpretation of the problem (Fig. 1).

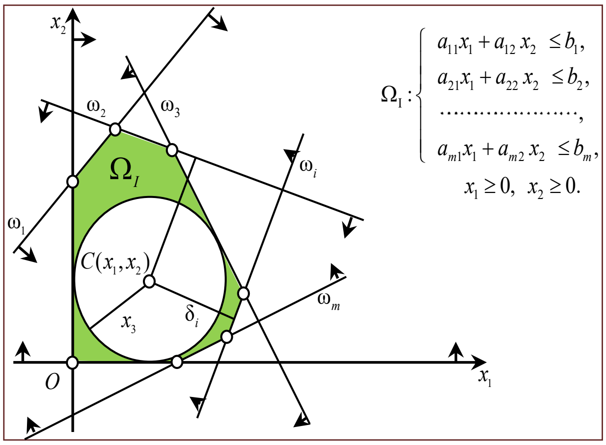

Fig. 1. Graphic interpretation of the problem

Let us have x_1, x_2 – as coordinates of the center of circle, and x_3 – as its radius. We introduce designations for boundaries of semi-planes provided by inequalities of the system $\omega_i : a_{i1}x_1 + a_{i2}x_2 = b_i$. The normal equation of boundaries of the semi-plane provided as $a_{i1}x_1 + a_{i2}x_2 \le b_i$ looks as follows:

$$\frac{-\mathrm{siqn}(b_i)a_{i1}}{\sqrt{a_{i1}^2+a_{i2}^2}}x_1+\frac{-\mathrm{siqn}(b_i)a_{i2}}{\sqrt{a_{i1}^2+a_{i2}^2}}x_2-p_i=0$$

We know that inserting coordinates of a point into a normal straight-line equation shall be its deviation δ from the straight line. As the circle center point $C(x_1,x_2)$ is to be located in semi-plane $a_{i1}x_1+a_{i2}x_2\le b_i$, its deviation

$$\delta=\frac{-\mathrm{siqn}(b_i)a_{i1}}{\sqrt{a_{i1}^2+a_{i2}^2}}x_1+\frac{-\mathrm{siqn}(b_i)a_{i2}}{\sqrt{a_{i1}^2+a_{i2}^2}}x_2-p_i$$

and the circle radius constitute the inequality

$$\delta+x_3\le 0\,.$$

We have the following statement: semi-plane $a_{i1}x_1+a_{i2}x_2\le b_i$ is sure to comprise a circle with center at point $C(x_1,x_2)$ and with radius x_3, in case of completing the inequality

$$a_{i1}x_1+a_{i2}x_2+x_3\sqrt{(a_{i1})^2+(a_{i2})^2}\le b_i\,.$$

In view of this, the problem on maximum-radius circle located in polyhedron shall be formulated as a linear optimization problem [3; 4; 9].

$$W_I=x_3\to\max$$

$$\Omega_I:\begin{cases}a_{11}x_1+a_{12}x_2\ \ +x_3\sqrt{(a_{11})^2+(a_{12})^2}\le b_1,\\ a_{21}x_1+a_{22}x_2\ \ +x_3\sqrt{(a_{21})^2+(a_{22})^2}\le b_2,\\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots,\\ a_{m1}x_1+a_{m2}x_2\ \ +x_3\sqrt{(a_{m1})^2+(a_{m2})^2}\le b_m,\\ -x_1+x_3\le 0,\\ -x_2+x_3\le 0,\end{cases}$$

$$x_1\ge 0,\ \ x_2\ge 0,\ \ x_3\ge 0.$$

Model example No. 1

Polyhedral region is specified by the system of linear inequalities

$$\Omega_I:\begin{cases}-5x_1+12x_2\ \le 60,\\ 7x_1+24x_2\ \le 168,\\ 4x_1-3x_2\ \le 24,\\ 12x_1+5x_2\ \le 120,\end{cases}$$

$$x_1\ge 0,\ \ x_2\ge 0.\quad .$$

We need to find coordinates of the center and the radius of the circle biggest in area and placed in region Ω_I .

According to the procedure proposed, we set up a linear optimization problem, i.e. a standard problem of linear optimization. The target function and the set of constraints shall look as follows:

$$W_I = x_3 \to \max$$

$$\Omega_I : \begin{cases} -5x_1 + 12x_2 + 13x_3 \le 60, \\ 7x_1 + 24x_2 + 25x_3 \le 168, \\ 4x_1 - 3x_2 + 5x_3 \le 24, \\ 12x_1 + 5x_2 + 5x_3 \le 120, \\ -x_1 + x_3 \le 0, \\ -x_2 + x_3 \le 0, \end{cases}$$

$$x_1 \ge 0,\ \ x_2 \ge 0,\ \ x_3 \ge 0.$$

For solving the problem using a simplex method, we move to canonical form of the problem [16]:

$$W_I = x_3 \to \max$$

$$\Omega_I : \begin{cases} -5x_1 + 12x_2 + 13x_3 + x_4 = 60, \\ 7x_1 + 24x_2 + 25x_3 + x_5 = 168, \\ 4x_1 - 3x_2 + 5x_3 + x_6 = 24, \\ 12x_1 + 5x_2 + 5x_3 + x_7 = 120, \\ -x_1 + x_3 + x_8 = 0, \\ -x_2 + x_3 + x_9 = 0, \end{cases}$$

$$x_i \ge 0,\ \ i = 1, 2, \ldots, 9.$$

Thus, we have an acceptable reference plan to start solving with use of the simplex method:

$$X_0 = [\ 0,\ 0,\ 0,\ |\ 60,\ 168,\ 24,\ 120,\ 0,\ 0\ ] \in \Omega_I$$

We carry out calculation by the common simplex method (Table 1).

The respective graphic interpretation of the solution is given on Fig. 2.

From the last simplex table, we obtain the optimum solution $x_1^{opt} = 3$, $x_2^{opt} = 3$ and $x_3^{opt} = 3$. Therefore, the maximum-radius circle that can be placed within the polyhedron has a center at point $C(\ 3,\ 3\ )$ and radius $R = 3$.

The proposed approach can be generalized on an arbitrary finite dimensionality problem. Let us, e.g., consider a four-dimensional case.

Table 1

			a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
Basis	*C*	*B*	0	0	1	0	0	0	0	0	0
a_4	0	60	–5	12	13	1	0	0	0	0	0
a_5	0	168	7	24	25	0	1	0	0	0	0
a_6	0	24	4	–3	5	0	0	1	0	0	0
a_7	0	120	12	5	13	0	0	0	1	0	0
a_8	0	0	–1	0	1	0	0	0	0	1	0
a_9	0	0	0	–1	1	0	0	0	0	0	1
Δ_j	$W_I(X_0)$ =	0	0	0	–1	0	0	0	0	0	0
a_4	0	60	8	12	0	1	0	0	0	–13	0
a_5	0	168	32	24	0	0	1	0	0	–25	0
a_6	0	24	9	–3	0	0	0	1	0	–5	0
a_7	0	120	25	5	0	0	0	0	1	–13	0
a_3	1	0	–1	0	1	0	0	0	0	1	0
a_9	0	0	1	–1	0	0	0	0	0	–1	1
Δ_j	$W_I(X_1)$ =	0	–1	0	0	0	0	0	0	1	0
a_4	0	60	0	20	0	1	0	0	0	–5	–8
a_5	0	168	0	56	0	0	1	0	0	7	–32
a_6	0	24	0	6	0	0	0	1	0	4	–9
a_7	0	120	0	30	0	0	0	0	1	12	–25
a_3	1	0	0	–1	1	0	0	0	0	0	1
a_1	0	0	1	–1	0	0	0	0	0	–1	1
Δ_j	$W_I(X_2)$ =	0	0	–1	0	0	0	0	0	0	1
a_4	0	0	0	0	0	1	–5/14	0	0	–15/2	24/7
a_2	0	3	0	1	0	0	1/56	0	0	1/8	–4/7
a_6	0	6	0	0	0	0	–3/28	1	0	13/4	–39/7
a_7	0	30	0	0	0	0	–15/28	0	1	33/4	–7,8571
a_3	1	3	0	0	1	0	1/56	0	0	1/8	3/7
a_1	0	3	1	0	0	0	1/56	0	0	–7/8	3/7
Δ_j	$W_I(X_3)$ =	3	0	0	0	0	1/56	0	0	1/8	3/7

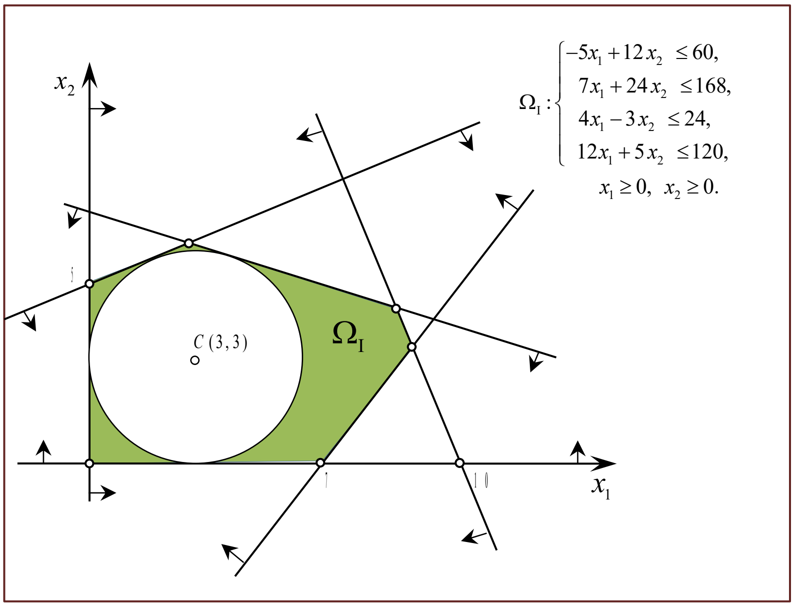

Fig. 2. Graphic representation of the biggest-radius circle inscribed in polyhedron Ω_I

Model example No. 2

The polyhedron is given by the system of linear inequalities:

$$\Omega_2 : \begin{cases} x_1 - 2x_2 + 4x_3 - 2x_4 \le 6, \\ 5x_1 + 6x_2 - 4x_3 - 2x_4 \le 2, \end{cases}$$
$$x_j \ge 0, \ j = 1,2,3,4.$$

We need to fine coordinates of the center and the radius of the circle biggest in area and placed in region Ω_2.

We set up a linear optimization problem respectively that shall look as follows:

$$W_I = x_5 \to \max$$
$$\Omega_2 : \begin{cases} x_1 - 2x_2 + 4x_3 - 2x_4 + 5x_5 \le 6, \\ 5x_1 + 6x_2 - 4x_3 - 2x_4 + 9x_5 \le 2, \\ -x_1 + x_5 \le 0, \\ -x_2 + x_5 \le 0, \\ -x_3 + x_5 \le 0, \\ -x_4 + x_5 \le 0, \end{cases}$$
$$x_j \ge 0, \ j = 1,2,3,4,5.$$

Canonical form of the linear optimization problem

$$\mathrm{W}_{\mathrm{I}} = x_5 \rightarrow \max$$

$$\Omega_2 : \begin{cases} x_1 - 2x_2 + 4x_3 - 2x_4 + 5x_5 + x_6 \;= 6, \\ 5x_1 + 6x_2 - 4x_3 - 2x_4 + 9x_5 + x_7 \;= 2, \\ -x_1 + x_5 + x_8 \;= 0, \\ -x_2 + x_5 + x_9 \;= 0, \\ -x_3 + x_5 + x_{10} \;= 0, \\ -x_4 + x_5 + x_{11} \;= 0, \end{cases}$$

$$x_j \geq 0,\; j = 1,2,\ldots,11.$$

Thus, we have an acceptable reference plan

$$X_0 = \left[\; 0,\; 0,\; 0,\; 0,\; 0 \mid\; 6,\; 2,\; 0,\; 0,\; 0,\; 0 \;\right] \in \Omega_2$$

We carry out calculation by the common simplex method. (Table 2).

Table 2

Basis	C	B	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}
			0	0	0	0	1	0	0	0	0	0	0
a_6	0	6	1	–2	4	2	5	1	0	0	0	0	0
a_7	0	2	5	6	–4	–2	9	0	1	0	0	0	0
a_8	0	0	–1	0	0	0	1	0	0	1	0	0	0
a_9	0	0	0	–1	0	0	1	0	0	0	1	0	0
a_{10}	0	0	0	0	–1	0	1	0	0	0	0	1	0
a_{11}	0	0	0	0	0	–1	1	0	0	0	0	0	1
Δ_i	$W_I(X_0) = 0$		0	0	0	0	–1	0	0	0	0	0	0
a_6	0	6	1	3	4	2	0	1	0	0	–5	0	0
a_7	0	2	5	15	–4	–2	0	0	1	0	–9	0	0
a_8	0	0	–1	1	0	0	0	0	0	1	–1	0	0
a_5	1	0	0	–1	0	0	1	0	0	0	1	0	0
a_{10}	0	0	0	1	–1	0	0	0	0	0	–1	1	0
a_{11}	0	0	0	1	0	–1	0	0	0	0	–1	0	1
Δ_i	$W_I(X_1) = 0$		0	–1	0	0	0	0	0	0	1	0	0
a_6	0	6	1	0	7	2	0	1	0	0	–2	–3	0
a_7	0	2	5	0	11	–2	0	0	1	0	6	–15	0
a_8	0	0	–1	0	1	0	0	0	0	1	0	–1	0
a_5	1	0	0	0	–1	0	1	0	0	0	0	1	0
a_2	0	0	0	1	–1	0	0	0	0	0	–1	1	0
a_{11}	0	0	0	0	1	–1	0	0	0	0	0	–1	1
Δ_i	$W_I(X_2) = 0$		0	0	–1	0	0	0	0	0	0	1	0

Table 2 (Continued)

Basis	C	B	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}
			0	0	0	0	1	0	0	0	0	0	0
a_6	0	6	1	0	0	9	0	1	0	0	–2	4	–7
a_7	0	2	5	0	0	9	0	0	1	0	6	–4	–11
a_8	0	0	–1	0	0	1	0	0	0	1	0	0	–1
a_5	1	0	0	0	0	–1	1	0	0	0	0	0	1
a_2	0	0	0	1	0	–1	0	0	0	0	–1	0	1
a_3	0	0	0	0	1	–1	0	0	0	0	0	–1	1
Δ_1	$W_1(X_3)$ =	0	0	0	0	–1	0	0	0	0	0	0	1
a_6	0	6	10	0	0	0	0	1	0	–9	–2	4	2
a_7	0	2	14	0	0	0	0	0	1	–9	6	–4	–2
a_4	0	0	–1	0	0	1	0	0	0	1	0	0	–1
a_5	1	0	–1	0	0	0	1	0	0	1	0	0	0
a_2	0	0	–1	1	0	0	0	0	0	1	–1	0	0
a_3	0	0	–1	0	1	0	0	0	0	1	0	–1	0
Δ_1	$W_1(X_4)$ =	0	–1	0	0	0	0	0	0	1	0	0	0
a_6	0	32/7	0	0	0	0	0	1	– 5/7	–18/7	–44/7	48/7	24/7
a_1	0	1/7	1	0	0	0	0	0	1/14	– 9/14	3/7	– 2/7	– 1/7
a_4	0	1/7	0	0	0	1	0	0	1/14	5/14	3/7	– 2/7	– 8/7
a_5	1	1/7	0	0	0	0	1	0	1/14	5/14	3/7	– 2/7	– 1/7
a_2	0	1/7	0	1	0	0	0	0	1/14	5/14	– 4/7	– 2/7	– 1/7
a_3	0	1/7	0	0	1	0	0	0	1/14	5/14	3/7	– 9/7	– 1/7
Δ_1	$W_1(X_5)$ =	1/7	0	0	0	0	0	0	1/14	5/14	3/7	– 2/7	– 1/7
a_{10}	0	2/3	0	0	0	0	0	7/48	– 5/48	– 3/8	–11/12	1	1/2
a_1	0	1/3	1	0	0	0	0	1/24	1/24	– 3/4	1/6	0	0
a_4	0	1/3	0	0	0	1	0	1/24	1/24	1/4	1/6	0	–1
a_5	1	1/3	0	0	0	0	1	1/24	1/24	1/4	1/6	0	0
a_2	0	1/3	0	1	0	0	0	1/24	1/24	1/4	– 5/6	0	0
a_3	0	1	0	0	1	0	0	3/16	– 1/16	– 1/8	– 3/4	0	1/2
Δ_1	$W_1(X_6)$ =	1/3	0	0	0	0	0	1/24	1/24	1/4	1/6	0	0

The last simplex table provides the optimum solution $_{\max}^{\text{opt}} \left[\frac{1}{3}, \frac{1}{3}, 1, \frac{1}{3}, \frac{1}{3}, 0, 0, 0, 0, \frac{}{3}, 0\right]$. The maximum-radius hypersphere that can be immersed into polyhedral region Ω_2 has its center at point $C\left(\frac{1}{3}, \frac{1}{3}, 1, \frac{1}{3}\right)$ and radius $R = \frac{1}{3}$.

Conclusions. Therefore, the project lifecycle modeling processes need to take account of a set of factors a part of which is represented in detail in PMI PMBOK standard. The proposed model based on a problem of placing a maximum-radius hypersphere in a polyhedron allows for taking account of the said factors and can be reduced to a classic linear optimization problem soluble by known methods. The polyhedron region is specified by a system of linear inequalities. In this case, the problem on the maximum-radius circle placed within the polyhedron is formulated as a linear optimization problem. Thus we can determine the maximum number of factors that must be taken into account while developing lifecycle of a certain project.

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